Essays questions

Lecture (21)

حل الأسئلة الكلامية ، دائما بكتب normaly distributed

(e.g) Suppose that the grades in a general examination are normaly distributed with mean 68 , Standard deviation equal to 10 , Find:-

(a) the proportion of grades that are more than 85.

(b) the proportion of grades that are between 60 & 90.

(c) the 95th percentile.

(Sol) $X \sim n(68, 100) \rightarrow \mu = 68 \;,\; \sigma = 10$

proportion = probability

(a) $p(X \geq 85)$

$p\left(Z \geq \frac{85-68}{10}\right) = p(Z > 1.7)$

$= 1 - p(Z \leq 1.7)$

$= 1 - 0.9554 = 0.0446$

(b) $p(60 < X < 90) = p\left(\frac{60-68}{10} < Z < \frac{90-68}{10}\right)$

$= p(-0.8 < Z < 2.2)$

$= p(Z < 2.2) - p(Z < -0.8)$

$= 0.9861 - 0.2119 = 0.7742$

(c) $P_{95} = ? \rightarrow p(X < P_{95}) = 0.95$

$p\left(Z < \frac{P_{95}-68}{10}\right) = 0.95$

$P_{95} = 84.40$

(e.g) If the height of students are normaly distributed with mean 170 cm & standard deviation 10 cm.

(i) a student is selected at random, what is the probability that this student is shorter than 175 cm?

(ii) 2 students are selected at random, what is the prob. that exactly 1 of them is shorter than 175 (with replacement)?

(iii) 10 student are selected one by one with replacement, what is the prob. that exactly 4 of them are shorter than 175 cm.

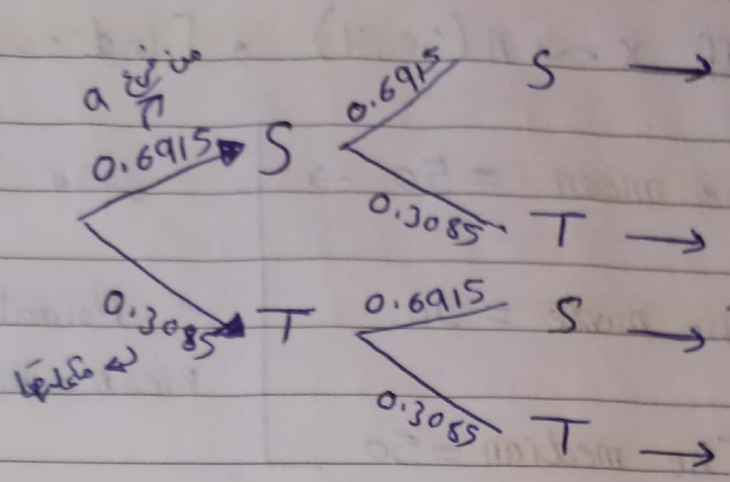
(Sol) X : height $\sim n(170, 10^2)$

(a) $p(x < 175) = p\left(z < \frac{175-170}{10}\right)$

$= p(z < 0.5) = 0.6915$

الحالة العادية

(b) على ال Tree diagram



$$(0.6915)(0.3085) + (0.3085)(0.6915) = \ldots$$

(c) → Binomial ← التوزيع ٣

Y: no. of students that are shorter than 175

$Y \sim Bin(10, 0.6915)$

n — احتمالية النجاح في المرة الواحدة

القانون القديم للبايونوميال (ممكن على الجدول للبايونوميال) ← $P(Y=4) = \binom{10}{4} \cdot (0.6915)^4 \cdot (0.3085)^6$

(e.g) If $X \sim n(50, 4)$, Find :-

(a) The mean $= 50 \rightarrow$

(b) The mode $= 50$

(c) The median $= 50$

مباشر

بما أنه Normal dis

إذا mean = mode = median

(d) The IQR

① $p(X < Q_1) = 0.25$

$p\left(Z < \frac{Q_1 - 50}{2}\right) = 0.25$

لـ من جدول السالب (أقرب قيم عليه)

$\frac{Q_1 - 50}{2} = 0.67$

$\therefore Q_1 = 48.7$

② $p(X < Q_3) = 0.75$

لـ وبنجل عادي (QR)

نستغل فكرة التوزيع المتماثل حيث أن :-

$Q_2 - Q_1 = Q_3 - Q_2$

$50 - 48.7 = Q_3 - 50$

$1.3 = Q_3 - 50$

$\therefore Q_3 = 51.3$

$IQR = 51.3 - 48.7 = 2.6$

(e) The 85th percentile.

$$P_{85} \rightarrow P(x < P_{85}) = 0.85$$

$$P\left(z < \frac{P_{85} - 50}{2}\right) = 0.85$$

← جدول الموجب (أقرب رقم عليه)

$$\frac{P_{85} - 50}{2} = 0.085$$

$$\therefore P_{85} = 52$$

(f) $\sigma^2 = 4$
$\sigma = 2$] → مباشرة