

(e.g) Suppose that the grades in a general examination are normally distributed with mean 68 & standard deviation of 12 points, a sample of 4 grades are to be drawn, what is the prob. that the average of the grades drawn will be:-

- (a) more than 71
 (b) less than 65.
 (c) between 66 & 74.

$$\text{(Sol)} \quad X_1, X_2, X_3, X_4 \sim N(68, 12^2)$$

$$\text{normal distribution average} \rightarrow \bar{X} \sim N(68, 6^2)$$

$$(6)^2 = 36 = \frac{12^2}{4}$$

$$(a) \quad P(\bar{X} > 71) = P(Z > 0.5) \leftarrow \text{تحويل } \bar{X} \text{ لـ } Z$$

$$= 1 - P(Z \leq 0.5) = 0.3085$$

$$(b) \quad P(\bar{X} < 65) = P(Z < -0.5) = 0.3085$$

$$(c) \quad P(66 < \bar{X} < 74) = P(-0.33 < Z < 1)$$

$$= P(Z < 1) - P(Z < -0.33)$$

$$= 0.8413 - 0.3707 = 0.4706$$

(eg) Suppose that the weight of orange boxes are normally distributed with mean 10 kgs & standard deviation of 1.5 kgs. If a no. of boxes will be loaded in a car with threshold 1000 kgs, find the no. of boxes that will be loaded so that their total weight doesn't exceed the threshold of the car with prob. about 0.95.

(Sol) $X_1, X_2, X_3, \dots, X_n \sim n(10, 1.5^2)$
 $\bar{X} \sim n(10, \frac{1.5^2}{n})$

total weight $\rightarrow P(\sum_{i=1}^n X_i \leq 1000) = 0.95$
 الى mean $\rightarrow P(\bar{X} \leq \frac{1000}{n}) = 0.95$
 طريقة القسمة على n
 موزونة ??

$$P(Z \leq \frac{1000/n - 10}{1.5/\sqrt{n}}) = 0.95$$

$\therefore \frac{1000/n - 10}{1.5/\sqrt{n}} = 1.64 \leftarrow$ من الجدول

تدخل على شكل معادلة تربيعية \leftarrow

let $\sqrt{n} = x \quad \therefore n = x^2$
 $x(\frac{1000}{x^2} - 10) = 2.46$
 $\frac{1000}{x} - 10x = 2.46$
 $1000 - 10x^2 = 2.46x$
 $x \sim 98$

x_1, x_2, \dots, x_n

Notes:- If $x_1, x_2, \dots, x_n \sim n(\mu, \sigma^2)$, then $\bar{x} \sim n(\mu, \frac{\sigma^2}{n})$ or $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim n(0,1)$, provided that σ is known.

If σ is unknown, then we estimate σ by

$$S = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} \quad ; \quad \text{then } T = \frac{\bar{x} - \mu}{S/\sqrt{n}} \sim$$

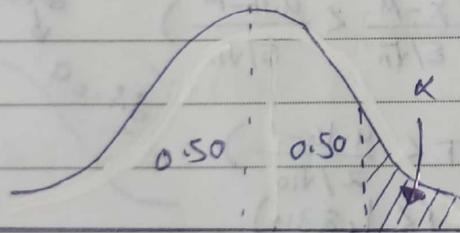
$t(n-1)$, t-distribution with $n-1$ degrees of freedom (d.f.).

But for $n \geq 30$, then $\frac{\bar{x} - \mu}{S/\sqrt{n}} = z \sim n(0,1)$

**** t - distribution :-**

الجدول خاص

(طريقة الاسترجاع)



1) نتجيب $df = n-1$

2) نبحث عن القيمة t في الجدول

سطر ال df

3) يتقاطع العمدة المطلوبة مع العمدة

سطر التي قيم مسيات t

على شكل t_α

حيث (α) هي مساحة المنطقة التي تقو ال (t)

← يعني النسبة الأخرى من t

↓ مساحة المنطقة الأخرى من (توتها)

(eg) Suppose that the weights of new born babies are normally distributed with mean 3 kgs. A random sample of size 10 is taken & showed that its standard deviation is 2.

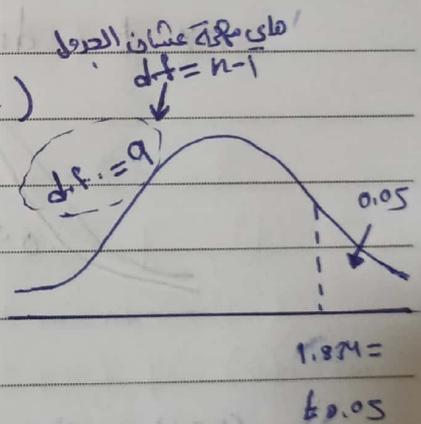
(a) Find the prob. that the sample average is below 4.16 kgs?

t-distribution ←

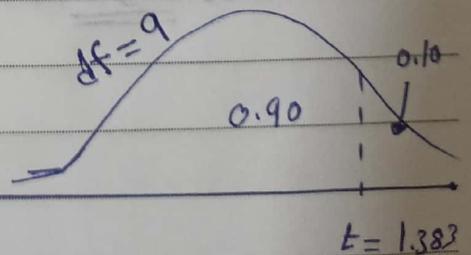
(b) What is the 90th percentile of the distribution of \bar{x} ?

(Sol) $X_1, X_2, \dots, X_{10} \sim N(3, \sigma^2)$
 $n=10, S=2$

$$\begin{aligned} (a) P(\bar{X} < 4.16) &= P\left(\frac{\bar{X} - \mu}{S/\sqrt{n}} < \frac{4.16 - \mu}{S/\sqrt{n}}\right) \\ &= P\left(T < \frac{4.16 - 3}{2/\sqrt{10}}\right) \\ &= P(T < 1.834) \\ &= 0.95 \end{aligned}$$



$$\begin{aligned} (b) P(\bar{X} < P_{90}) &= 0.90 \\ P\left(T < \frac{P_{90} - 3}{2/\sqrt{10}}\right) &= 0.9 \end{aligned}$$



$$\therefore \frac{P_{90} - 3}{2/\sqrt{10}} = 1.383$$

$$\therefore P_{90} = 3.87$$

(e.g) If $T \sim t(10)$, find :-

درجة df ↑

(a) the 95th percentile of T.

(b) the 10th percentile of T.

(Sol) d.f = 10

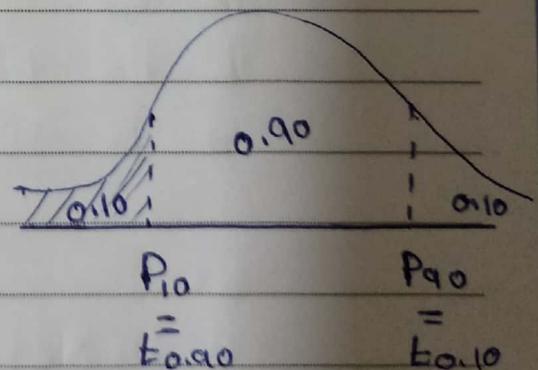
$$(a) P(T \leq P_{95}) = 0.95 \rightarrow t_{0.05}$$

$$\therefore P_{95} = 1.812$$

$$(b) P(T < P_{10}) = 0.10 \rightarrow t_{0.90} \rightarrow \text{الجدول لا يعطي المناطق السالبة. (معناها لازم نوظف فكرة التماثل)}$$

$$P_{10} = -(P_{90})$$

$$\therefore P_{10} = -1.322$$



a → 1.812