** The distribution of the sample variance S^2:- *توزيع الـvariance

* If $x_1, x_2 \dots x_n \sim n(\mu, \sigma^2)$, then

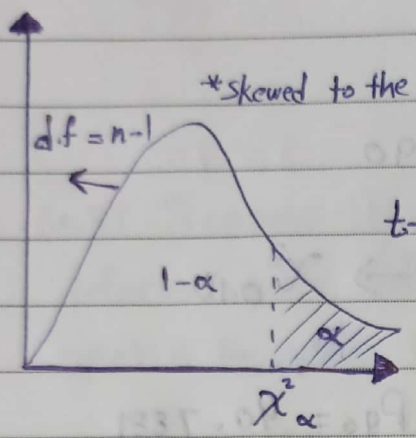
$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

↓

Chi-squared

d.f. $= n-1$

*skewed to the right.



χ^2- distribution نفس طريقة استخدام الـ t-distribution

(e.g) If a sample of size $n=6$ is drawn from a population with variance $\sigma^2 = 10$. Find $P(S^2 > 18.4727)$

(Sol) $P\left(\frac{S^2(n-1)}{\sigma^2} > \frac{(n-1)(18.4722)}{10}\right)$

$P(\chi^2 > 9.2364) = 0.10$

(e.g) Let $x_1, \dots, x_{10} \sim n(\mu, 25)$. If s^2 is the Sample variance, find the 90th percentile of s^2.

(Sol) $P(s^2 < P_{90}) = 0.90$

$$P\left(\chi^2 < \frac{(n-1)P_{90}}{\sigma^2}\right) = 0.90$$

$$P\left(\chi^2 < \frac{9 \cdot P_{90}}{25}\right) = 0.90 \rightarrow \chi^2_{0.10}$$

? $\frac{9 \cdot P_{90}}{25} = 14.6437 \quad \therefore P_{90} = 40.7881$

(e.g) Let $X \sim \chi^2(10)$. Find :-

(a) the 10th percentile of X.
(b) the 95th percentile of X.
(c) the 99th percentile of X.

(Sol)

(a) $P(X < P_{10}) = 0.10 \rightarrow \chi^2_{0.90}$

$\therefore P_{10} = 4.87$

(b) $P(X < P_{95}) = 0.95 \rightarrow \chi^2_{0.05}$

$\therefore P_{95} = 18.3070$

(c) $P(X < P_{99}) = 0.99 \rightarrow \chi^2_{0.01}$

$\therefore P_{99} = 23.2093$

** The distribution of the sample proportion $\hat{p}$:-

$$\hat{p} \sim n\left(p, \frac{pq}{n}\right)$$

or

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim n(0,1)$$

(e.g) Suppose that 10% of a certain production are defective. If 400 items are drawn from the production, what is the prob. that the sample proportion will be :-

(a) more than 12%

(b) between 9% & 11%

(Sol) $p = 0.10$, $q = 0.90$, $n = 400$

$$\hat{p} \sim n\left(0.10, \frac{(0.10)(0.90)}{400}\right)$$

$$\therefore \hat{p} \sim n\left(0.10, \left(\frac{3}{200}\right)^2\right)$$

(a) $P(\hat{p} > 0.12) = P(z > 1.33)$

$= 1 - P(z \leq 1.33) = 1 - 0.9082 = 0.0918$

(b) $P(0.09 < \hat{p} < 0.11) = P(-0.67 < z < 0.67)$

$= 0.7486 - 0.2514 = 0.4972.$

(e.g) Suppose that 90% of the university students pass Calculus 101. In a sample of 200 students taking Calculus 101, what is the prob. that the proportion of those who will pass is less than 85%.

(Sol) $p = 0.90$, $q = 0.10$, $n = 200$

$$\hat{p} \sim n\left(0.90, \frac{(0.90)(0.10)}{200}\right)$$

$$P(\hat{p} < 0.85) = ?$$

$$P\left(z < \frac{0.85 - 0.90}{\sqrt{\frac{9}{2000}}}\right)$$

$$\therefore P(z < -2.36) = 0.0091$$

To sum up :-

Population	Sample
↓	↓
parameters	estimators
(1) mean (μ)	(1) mean ($\bar{x}$)
(2) Variance (σ^2)	(2) Variance (s^2)
(3) proportion (p)	(3) proportion ($\hat{p}$)

توزيع لكل مقدار منهم :-

$\boxed{1}$ $\bar{x} \sim n(\mu, \frac{\sigma^2}{n}) \rightarrow \sigma$ known

$\bar{x} \sim t(n-1) \rightarrow \sigma$ unknown , $n < 30$

$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim n(0,1) \rightarrow \sigma$ unknown , $n \geq 30$

$\boxed{2}$ $\frac{(n-1) \cdot s^2}{\sigma^2} \sim \chi^2 (n-1)$

$\boxed{3}$ $\hat{p} \sim n(p, \frac{pq}{n})$