

## Lecture (26)

The distribution of the difference between 2 sample means :-

If  $x_1, \dots, x_n \stackrel{r.s}{\sim} n(\mu_1, \sigma_1^2)$  &  
 $y_1, \dots, y_m \stackrel{r.s}{\sim} n(\mu_2, \sigma_2^2)$ , then

$$\bar{x} \sim n(\mu_1, \frac{\sigma_1^2}{n})$$

$$\bar{y} \sim n(\mu_2, \frac{\sigma_2^2}{m})$$

$$\therefore \bar{x} - \bar{y} \sim n(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m})$$

provided that  $\sigma_1$  &  $\sigma_2$  are known

$$\text{or } Z = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim n(0, 1)$$

If  $\sigma_1 = \sigma_2 = \sigma$  (unknown), then

$$T = \frac{(\bar{x} - \bar{y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{(S_p^2)}{n} + \frac{(S_p^2)}{m}}} \sim t(n+m-2)$$

$$\text{where } S_p^2 = \frac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$$

is the pooled variance.

Note :- If  $n, m \geq 30$ , then

$$\frac{\bar{Z} = (\bar{X} - \bar{Y}) - (M_1 - M_2)}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}} \sim N(0, 1)$$

(e.g) Suppose that the grade of female and male students in Calculus 101 are normally distributed with means  $70 \pm 65$ , respectively & standard deviations  $8 \pm 10$ , respectively. In Samples of (15) female & 20 male students, find the prob. that the female students will have an average more than male students average?

$$(Sol) X_1, X_2, \dots, X_{15} \sim N(70, 8^2)$$

$$Y_1, Y_2, \dots, Y_{20} \sim N(65, 10^2)$$

$$\bar{X} \sim N\left(70, \frac{8^2}{15}\right)$$

$$\bar{Y} \sim N\left(65, \frac{10^2}{20}\right)$$

$$\bar{X} - \bar{Y} \sim N\left(5, \frac{8^2}{15} + \frac{10^2}{20}\right)$$

تابع

$$\begin{aligned} P(\bar{x} > \bar{y}) &= P(\bar{x} - \bar{y} > 0) \\ &= P\left(Z > \frac{0-5}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}}\right) = P(Z > -1.64) \\ &= 1 - P(Z \leq -1.64) = 0.9495 \end{aligned}$$

The distribution of the difference between 2 sample proportions :-

$$\hat{P}_1 - \hat{P}_2 \sim n\left(P_1 - P_2, \frac{P_1 q_1}{n} + \frac{P_2 q_2}{m}\right)$$

$$\text{or } Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n} + \frac{P_2 q_2}{m}}} \sim N(0, 1)$$

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(eg) Suppose that 50% of population (a) own cars while 35% of population (b) own cars. If a sample of size 100 is drawn from population (a) & a sample of size 80 is drawn from population (b). What is the prob. that the difference between the proportion  $\hat{P}_A - \hat{P}_B$  will be between 0.1 & 0.2?

$$(\text{Sol}) \quad \underline{A} \quad P_1 = 0.50, \quad n = 100, \quad q_1 = 0.50$$

$$\underline{B} \quad P_2 = 0.35, \quad m = 80, \quad q_2 = 0.65$$

$$\hat{P}_1 - \hat{P}_2 \sim n \left( 0.5 - 0.35, \frac{(0.5)(0.5)}{100} + \frac{(0.35)(0.65)}{80} \right)$$

$$\hat{P}_1 - \hat{P}_2 \sim n(0.15, (0.073)^2)$$

$$\hookrightarrow P(0.1 < \hat{P}_1 - \hat{P}_2 < 0.2)$$

$$P\left(\frac{0.1 - 0.15}{0.073} < \frac{\hat{P}_1 - \hat{P}_2}{0.073} < \frac{0.2 - 0.15}{0.073}\right)$$

$$= P(-0.68 < Z < 0.68)$$

$$= P(Z < 0.68) - P(Z < -0.68) = 0.5034.$$

$$(A-A) - (A-A) = 0$$

$$\frac{IP_A}{n} + \frac{IP_B}{n}$$

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