

## Concepts of statistical Inference

\* Confidence Interval

Tests of hypotheses

\* Estimation by C.I. :-

Let  $L \leq T$  be functions of  $x_1, x_2, \dots, x_n$ :

$(L, T)$  is  $(1-\alpha) 100\%$  C.I. ( $0 < \alpha < 1$ ) for  $\theta$  if &  
 $P(L < \theta < T) = 1 - \alpha$

(1)  $1 - \alpha$  :- confidence coefficient.

(2)  $\alpha$  :- significance level.

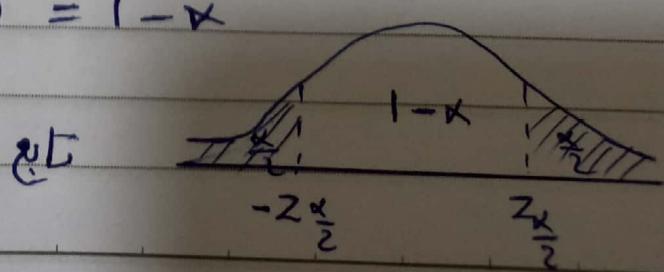
(3)  $L$  :- lower confidence limit (L.C.L.)

(4)  $T$  :- upper confidence limit (U.C.L.)

Note:- If  $x_1, x_2, \dots, x_n \sim n(\mu, \sigma^2)$ , then

$$\bar{x} \sim n(\mu, \frac{\sigma^2}{n}) \text{ or } Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim n(0, 1)$$

$$P(-\frac{z_{\alpha/2}}{2} < Z < \frac{z_{\alpha/2}}{2}) = 1 - \alpha$$



(T.C) (not true)

(I.S) (true)

$$P\left(-\frac{Z_{\alpha/2}}{2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{Z_{\alpha/2}}{2}\right) = 1-\alpha$$

$$P\left(-\frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}} < \bar{X}-\mu < \frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

$$P\left(-\bar{X}-\frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < -\bar{X} + \frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

$$\therefore P\left(\bar{X}-\frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + \frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

\*\* the  $(1-\alpha)100\%$  C.I. for  $\mu$  is :-

$$\left(\bar{X}-\frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + \frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$\downarrow$

L

$\downarrow$

U

I The C.I. for  $\mu$  :-

the  $(1-\alpha)100\%$  C.I. for  $\mu$  is :-

(i)  $\bar{X} \pm \frac{Z_{\alpha/2}}{2} \cdot \frac{\sigma}{\sqrt{n}}$ , if  $\sigma$  is known.

(ii)  $\bar{X} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ , if  $\sigma$  is unknown,  $n < 30$

(iii)  $\bar{X} \pm Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ , if  $\sigma$  is unknown,  $n \geq 30$

Note 2  $\frac{Z}{2} \cdot \frac{\sigma}{\sqrt{n}} : \text{Error} = E$

$$\frac{\sigma}{\sqrt{n}} : \text{Standard error (S.E.)}$$

(e.g) The salaries of teachers in Jordan for 1990-2000 are normally distributed with standard deviation 50 JD. The average salary based on a sample of 400 teachers for 1990-2000 was 215 JD per month

(a) What is the point estimate for the mean salaries & its S.E.?

\* point estimate for  $\mu$  is  $\bar{x} = 215$

\* standard error = S.E.

$$= \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{400}} = 2.5$$

(b) Give a 90% C.I. for the mean salaries.

$$1 - \alpha = 0.90 \Rightarrow \alpha = 0.10$$

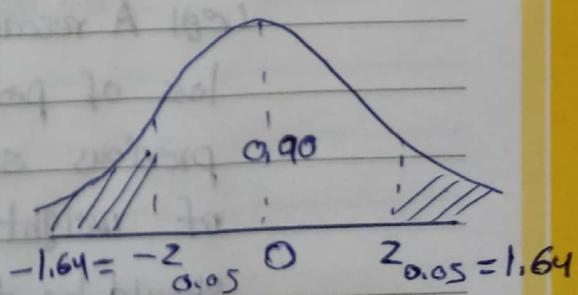
$$\frac{\alpha}{2} = 0.05$$

The 90% C.I. for  $\mu$  is

$$215 \pm (1.64) \cdot \left( \frac{50}{\sqrt{400}} \right)$$

$$L = 210.90$$

$$U = 219.10$$



(c) Give a 95% C.I. for the mean salaries

$$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$$

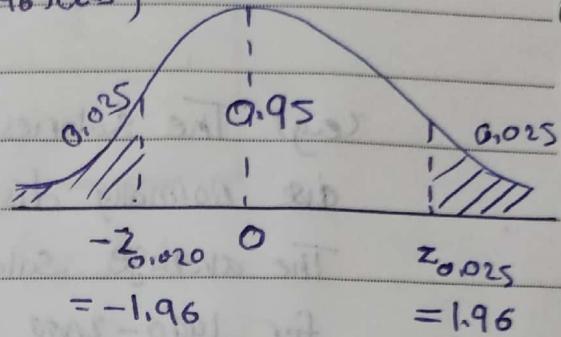
$$\therefore \frac{\alpha}{2} = 0.025$$

$$L = \bar{x} - \frac{z_{0.025} \cdot \frac{6}{\sqrt{n}}}{0.025} = 215 - (1.96)(0.5)$$

$$\therefore L = 210.10$$

$$U = \bar{x} + \frac{z_{0.025} \cdot \frac{6}{\sqrt{n}}}{0.025}$$

$$\therefore U = 219.90$$



$\therefore$  The 95% C.I. for  $\mu$  is:-

$$(210.10, 219.90)$$

\*\* Sample size to estimate  $\mu$  :-

$$n = \left( \frac{z_{\alpha/2}}{E} \right)^2 \cdot \sigma^2 \rightarrow$$

الخطوات

(c) A researcher wants to estimate the average weight loss of people who are on a new diet plan. In a previous study, the population standard deviation ( $\sigma$ ) of weight losses is about 5 kgs. How large a sample should be to estimate the mean weight loss by a 95% C.I. to within 1.5 kgs?

$$\sigma = 5$$

$$n = ??$$

$$1 - \alpha = 0.95 \rightarrow \alpha = 0.05$$

$$\therefore \frac{\alpha}{2} = 0.025$$

$$z_{0.025} = 1.96$$

$$n = \left( \frac{z_{\alpha/2}}{E} \right)^2 \cdot \sigma^2$$

$$\therefore n = 42.68 \approx 43$$

الإجابة  $\leftarrow$

(e.g) Suppose that Jordan Bureau of  
Wants to estimate the mean size ( $\mu$ )