

## Lecture (28)

also  $\sigma$  cuts all is also \*\*

(eg) The mean cholesterol levels in a general population are normally distributed. A sample of (16) persons is taken under a test with sample mean  $\bar{X} = 220$  & standard deviation  $S = 25$ . Give a 90% C.I for the population mean  $\mu$ .

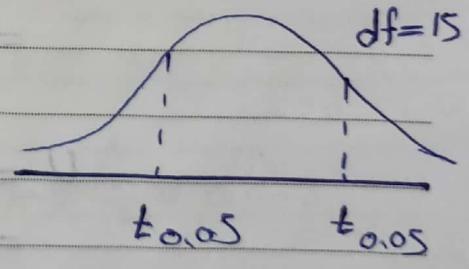
$$(\text{Sol}) \quad n = 16, \quad \bar{X} = 220, \quad S = 25$$

$$30 > n \quad (\text{r}) \quad \text{also } \sigma \quad (\text{l})$$

$$1 - \alpha = 0.90 \rightarrow \alpha = 0.10 \quad \therefore \frac{\alpha}{2} = 0.05$$

$$\bar{X} \pm t_{0.05} \cdot \frac{S}{\sqrt{n}}$$

$$220 \pm (1.753) \left( \frac{25}{4} \right)$$



$$\therefore L = 209.04$$

$$\therefore U = 230.96$$

م من العدد الـ  $t$  الـ  $t_{0.05}$   $\leftarrow$   
،  $Z$  الـ  $t_{0.05}$

The 90% C.I. for  $\mu$  is:-

$$(209.04, 230.96)$$

(e.g) A random sample of 400 people with a professional degree taken showed that their mean monthly salary is 450 JD with a standard deviation of 100 JD. Give a 90% C.I for the mean monthly salary.

$$(\text{Sol}) \quad n = 400 \quad , \quad \bar{X} = 450 \quad \Rightarrow \quad S = 100$$

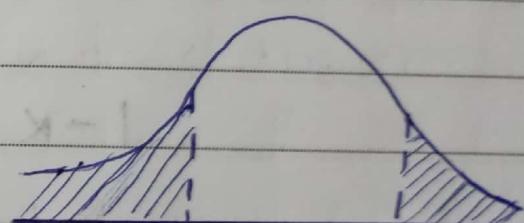
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$$1 - \alpha = 0.90 \rightarrow \alpha = 0.10 \rightarrow \frac{\alpha}{2} = 0.05$$

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}$$

$$L = 450 - (1.64) \left( \frac{100}{\sqrt{400}} \right)$$

$$= 441.8$$



$$U = 450 + (1.64) \left( \frac{100}{\sqrt{400}} \right)$$

$$= 458.2$$

The 90% C.I. for  $\mu$  is:-

$$(441.8, 458.2)$$

## 2 Interval Estimation for $P$ :-

The  $(1-\alpha) 100\%$  C.I. for  $P$  is :

$$\hat{P} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$$

(e.g) It was believed in the Arab world that 50% of persons are smoking. During the year 2000, a sample of 1000 persons showed that the no. of smokers is (620). Establish 95% C.I. for the proportion of smokers

$$(\text{Sol}) \quad n = 1000 \quad , \quad X = 620 \quad ,$$

$$\hat{P} = \frac{X}{n} = \frac{620}{1000} = 0.62$$

$$\hat{q} = 1 - 0.62 = 0.38$$

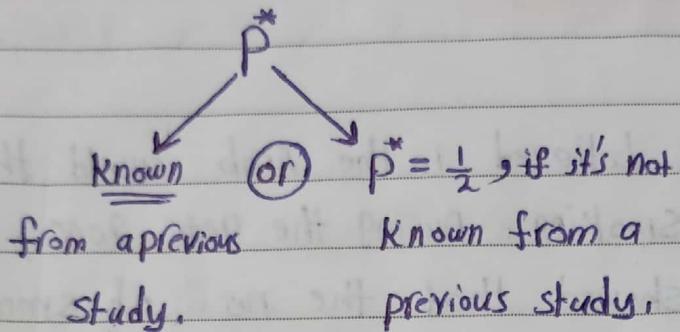
$$1 - \alpha = 0.95 \rightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$$

$$\frac{Z_{\alpha/2}}{2} = 1.96 \rightarrow$$

$$\rightarrow \hat{P} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{P}\hat{q}}{n}}$$

## \* Determination of the sample size :-

$$n = \left( \frac{Z_{\alpha/2}}{E} \right)^2 \cdot p^* \cdot (1-p^*)$$



(e.g) Assume that it is required to estimate the proportion of patients suffering a bad reaction from taking a reaction medication  $p$  by 95% C.I. Determine the sample size needed if the error estimation is about 0.10 in the following cases :-

(a) no prior information about  $p$ .

(b) Previous study showed that  $p$  is approximately .

$$(sol) \quad n = ? , E = 0.10$$

$$1 - \alpha = 0.95 \rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$$

$$Z_{0.025} = 1.96$$

$$(a) \quad p^* = 0.5$$

$$n = \left( \frac{1.96}{0.10} \right)^2 \cdot (0.5)(1-0.5) \approx 97$$

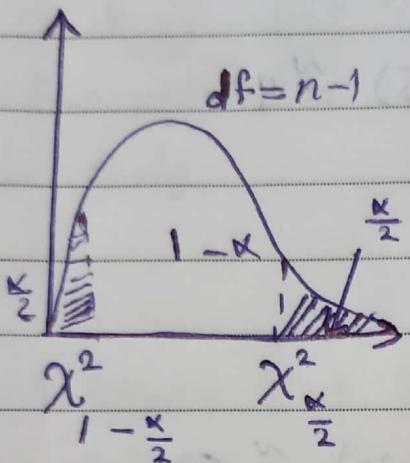
$$(b) \quad p^* = 0.20$$

$$n = \left( \frac{1.96}{0.10} \right)^2 (0.20)(1-0.20) \approx 62$$

### **3 Interval Estimation for $\sigma^2$ :-**

The  $(1-\alpha) 100\%$  C.I. for  $\sigma^2$  is:

$$\frac{(n-1)s^2}{s^2} \sim \chi^2(n-1) \Rightarrow \text{ما نعم شرط سابقا}$$



$$\left( \frac{(n-1)S^2}{\chi_{\frac{\alpha}{2}}^2}, \frac{(n-1)S^2}{\chi_{1-\frac{\alpha}{2}}^2} \right)$$

(e.g) Quality - central engineer wishes to study the weight variation of anew product . a sample of 10 items is taken & provided that  $\bar{X}=0.60$  kgs &  $S=0.4$  kgs  
 Find the 90% C.I. for the variances of all items.  
 (Assume that the distribution of the weight can be modeled as anormal distribution )

$$(Sol) \quad n=10, \bar{x}=0.6 \rightarrow S=0.40$$

$$1 - \alpha = 0.90 \rightarrow \alpha = 0.10 \rightarrow \frac{\alpha}{2} = 0.05$$

$$\therefore \frac{(10-1)(0.4)^2}{\chi^2_{0.05}} \quad , \quad \frac{(10-1)(0.4)^2}{\chi^2_{0.95}}$$

← من العدد

$$= (0.09, 0.43)$$