Chapter 10: Fluids

- Fluid statics: sections 10.1 → 10.7: fluids at rest (included).
- Fluid dynamics: sections 10.8 → 10.10: fluids in motion (excluded).

I Sections 10.1 → 10.3: Density (ρ) and Pressure (P).

- A fluid, in contrast to a solid, is a substance that can flow. Thus we lump liquids and gases together and call them fluids.
- When we discuss solid (rigid) object, we are concerned with particular lumps of matter, such as blocks or rods. Physical quantities that we find useful, and in whose terms we express Newton's 2nd law, are mass and force. With fluids, where mass and volume can vary, it is more useful to speak of density and pressure than of mass and force.
- Take gold as a substance and take ring and necklace, both made of gold, as objects:

→ the mass of ring ≠ the mass of necklace

→ the density of ring = the density of necklace

⇒ Density = mass per unit volume;

$\rho = \frac{m}{V}$ → scalar quantity, its SI unit: $\frac{kg}{m^3}$

∴ ρ is an intrinsic property of the substance.

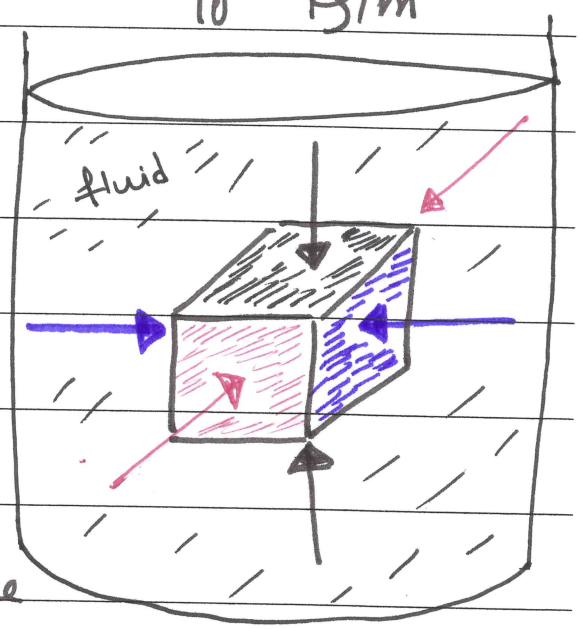
- Definition: Specific Gravity of a substance (x):

$$SG(x) = \frac{\rho(x)}{\rho(H_2O)} \quad \text{——— (1)}$$

⇒ Table 10:1 ⇒ $SG(\text{lead} - Pb) = \frac{11.3\,10^3\ kg/m^3}{10^3\ kg/m^3} = 11.3$

⇒ SG = pure number.

- At any point on the surface of an object, the force exerted by the fluid is perpendicular to the surface of the submerged object. If F is the magnitude of the force and A is the surface area, the pressure P of the fluid on the submerged object is defined as



$$P \equiv \frac{F}{A} \quad \text{——— (2)}$$

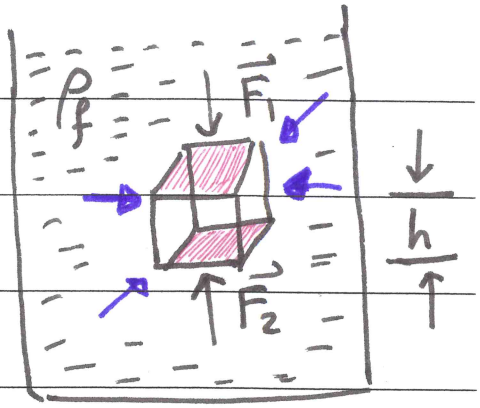
- The definition (eq 2) makes a clear distinction between F and P. $\vec{F}$ is a vector and P is a scalar. There is no direction associated with P, but the direction of F associated with P is ⊥ to the surface on which the pressure acts. The SI unit of P is N/m^2, which has been named the pascal, abbreviated $Pa \equiv 1\,N/1\,m^2$ ——— (3).
- Recall the dent in the carpet caused by the table legs in lecture 4 page 7 !!
- A PHY 105 standing on one foot ⇒ $P = mg/A$

  " " " " " two feet ⇒ $P = mg/2A$

  " " " " " one foot "with closed eyes" ⇒ P ?
  "neurological function".
- Do example 10.2.
- Pressure – depth relationship:

As divers well know, water pressure increases with depth, resulting in a feeling of discomfort in the ears of the diver. We now show details of how the pressure in a fluid increases with depth.

Consider a tank of fluid and imagine a cube of the fluid inside of it. The fluid pressure acting on the cube produces forces.



However, by Newton's 1st law, there must be no net force acting on this <u>stationary</u> cube of fluid. The forces acting on the vertical sides of the cube (⇄ ⇅) clearly cancel out. The vertical forces acting on the bottom and top sides (▨) of the cube must also add to zero:

$$F_2 - F_1 - mg = 0 \quad \text{——} \quad (4)$$

* F_1 is the force downward on the top of the cube,
* F_2 is the force upward on the bottom of the cube,
* mg is the weight of the cube of fluid (ρ_f).

⇒ Assuming that the area of the top and bottom surfaces of the cube is A, we can write the forces F_1 and F_2 in terms of the pressures P_1 and P_2 as:

$F_1 = P_1 A$ and $F_2 = P_2 A$. $\Rightarrow$ (recall of 2).

$\Rightarrow$ We can also express the mass, m, of the fluid in terms of the density of fluid, ρ_f, and the volume, V, of the cube, $m = \rho_f V$. Substituting for F_1, F_2, and m in (4):

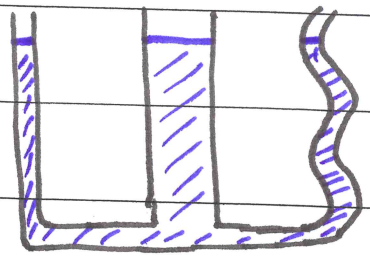
$$P_2 A - P_1 A - \rho_f V g = 0.$$

let $V = A \times h$, where h is the depth of the cube (page 4).

$$\Rightarrow \quad P_2 \cancel{A} = P_1 \cancel{A} + \rho_f \cancel{A} h g$$

$$\therefore \boxed{P_2 = P_1 + \rho_f g h} \quad \text{—— (5)}$$

Eq (5) specifies the pressure in a fluid at a vertical depth of h, without any dependence on horizontal position. Note that the area A is cancelled out, and thus does not affect the pressure at a given depth. Thus, the equation holds regardless of the shape of the vessel containing the fluid. Exercise A shows different containers of the same fluid (ρ_f). Check it out!



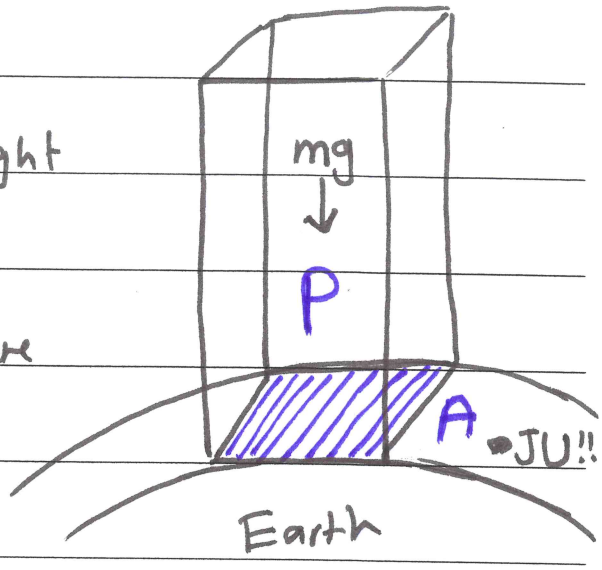
- This example shows 3 connected columns containing a fluid (ρ_f). You can see that the fluid reaches the same height in each column, independent of the shape or cross-sectional area; this occurs because the bottoms are interconnected and thus have the same pressure.

II Section 10.4 and Section 10.6:

Atmospheric pressure and Gauge pressure:

- The atmospheric pressure, P_{atm}, is, as the name suggests, the approximate average pressure of the atmosphere. It seems rather daunting to determine P_{atm}, because the density of air varies greatly with altitude, and there is no distinct top surface to the atmosphere from which h (in eq 5) could be measured.

The sketch shows a column of the earth's atmosphere with weight mg above an earea A of earth's surface. This air exerts a pressure P on the surface area of earth.



$$P = \frac{F}{A} = \frac{mg}{4\pi R_E^2}.$$

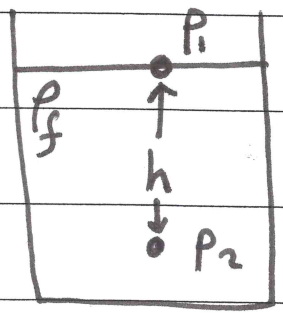
- The average pressure of the earth's atmosphere at sea level, 1 atm, is a commonly used non-SI unit that is expressed in other units as follows:

$$1 \text{ atm} = 1.013\, 10^5 \text{ Pa} = 1.013 \text{ bar} \quad \text{—— (6)}, \text{ where}$$

$$1 \text{ bar} = 1.000\, 10^5 \text{ Pa} \quad \text{—— (7)}.$$ Check Table 10.2.

- Recall eq(5)

$$P_2 = P_1 + \rho_f g h$$



If the fluid is open to the atmosphere, then P_1 is the atmospheric pressure; P_{atm} (1 atm).

- The pressure P_2 is called the absolute pressure, and the difference $P_2 - P_{atm}$ is called the gauge pressure (because we use a gauge to measure this pressure difference): $P_{absolute} = P_{atm} + P_{gauge}$

$$\therefore P_{absolute} = P_{atm} + \rho_f g h \quad \text{——} \quad (8)$$

The absolute pressure includes the pressure of the fluid as well as the pressure of the air (atmosphere) above it.

- The gauge pressure is $\rho_f g h$ as given in eq 8. In many cases we are intrested in the gauge pressure. For example, a flat tire doesn't have zero pressure in it; the pressure in the tire is P_{atm}. To inflate the tire to 241 kPa [Table 10.2: 1 lb/in^2 = 6.9 kPa, thus 241 kPa $\cong$ 35 lb/in^2], the absolute pressure inside the tire must be greater than P_{atm} by this amount; that is, $P = 241\ kPa + 1\ atm = 342.7\ kPa$.

It is the gauge pressure that is determined by a tire gauge (35 lb/in^2 as an example).

- The gauge pressure can be, by definition, positive or negative, depending on whether $P_{absolute} > P_{atm}$ or $P_{absolute} < P_{atm}$. In inflated tires or the human circulatory system (as we will discuss on page 14), the absolute pressure is greater than the atmospheric pressure, so the gauge pressure is a positive quantity. If you suck on a straw to pull fluid up the straw, the absolute pressure in your lungs is actually less than atmospheric pressure. The gauge pressure in your lungs is then a negative quantity.

- Example: A pain in your ear:

Estimate the force exerted on your eardrum due to the water when you are swimming at the bottom of a pool that is 5 m deep. Assume that the surface area of the eardrum is approximately $1\,cm^2$.

$\Rightarrow$ The air inside the middle ear is normally at P_{atm}. Therefore, to find the net force on the eardrum, we

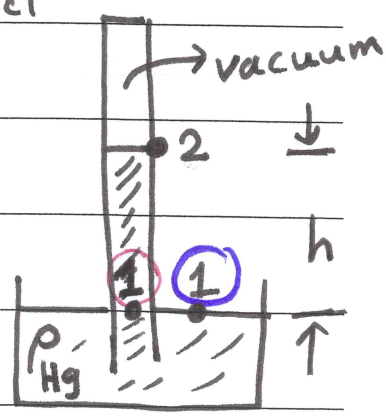
must consider the difference between the absolute pressure at the bottom of the pool and P_{atm}, that is the gauge pressure.

$$P_{gauge} = \rho_f g h = 10^3 * g * 5 = 4.9\,10^4 \text{ Pa}$$

$$\Rightarrow \quad F = P_{gauge} * A = 4.9\,10^4 * 10^{-4} \text{ m}^2 \simeq 5\text{N}.$$

Because a force of this magnitude on the eardrum is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort. ▪ Do Problem 10.71 : pop your ear!

▪ As we have seen, it is quite difficult to calculate precisely P_{atm}. A simple device used to measure P_{atm} is the mercury barometer. You can construct a mercury barometer by taking a long tube, closed at one end, filling it with mercury, and inverting it with the open end in a dish of mercury.



The space above the mercury is a vacuum (void) and thus has zero pressure. The pressure at point 1, due to the column of mercury, must equal the pressure at point 1, due to the atmosphere: [If that were not the case, there would be a net force that would move mercury – in contradiction to our assumption that the fluid is at rest.]. Therefore, the difference in height between the top of the mercury in the tube (2) and the top of the mercury in the dish (1) $\equiv h \equiv$, can be related to P_{atm} using eq (8) $\equiv$ eq (5):

$$P_2 = 0, \; P_1 = P_{atm} \Rightarrow P_1 = P_2 + \rho_{Hg} g h$$

$$\therefore P_{atm} = \rho_{Hg} g h.$$

- Let's determine the height of a mercury column for 1 atm of pressure:

$$h = \frac{P_{atm}}{\rho_{Hg}\, g} = \frac{1.013 \times 10^5 \text{ Pa}}{13.6 \times 10^3 \frac{kg}{m^3} \, 9.8 \, m/s^2} = 0.760 \, m.$$

Table 10.1

Based on such a calculation, 1 atm of pressure is defined to be the pressure equivalent of a column of mercury

that is exactly 0.760 m in height at 0°C.

- The unit mm-Hg is called the torr, in honor, of Torricelli, who invented the barometer.

$1\ \text{mm-Hg} = 1\ \text{torr} = \rho_{Hg} * g * (h = 1\text{mm}) = 133\ \text{Pa}$ — (9)

$760\ \text{mm-Hg} = 760\ \text{torr} = 1\ \text{atm} = 1.013\,10^{5}\ \text{Pa} = 1.013\ \text{bar}$ — (10)

- That being said, there is nothing "sacred" about mercury! If the barometer is filled with water, a similar calculation shows that the atmospheric pressure can support (maintain) a column of water 10.3 m high in the tube whose closed top is under vacuum:

the height h which can be calibrated to measure 1 atm for water is $h = \dfrac{P_{atm}}{\rho_{H_2O}\, g} = \dfrac{1.013\,10^{5}\ \text{Pa}}{10^{3}\ \text{kg m}^{-3} \cdot 9.8\ \text{m s}^{-2}} =$ 10.3m

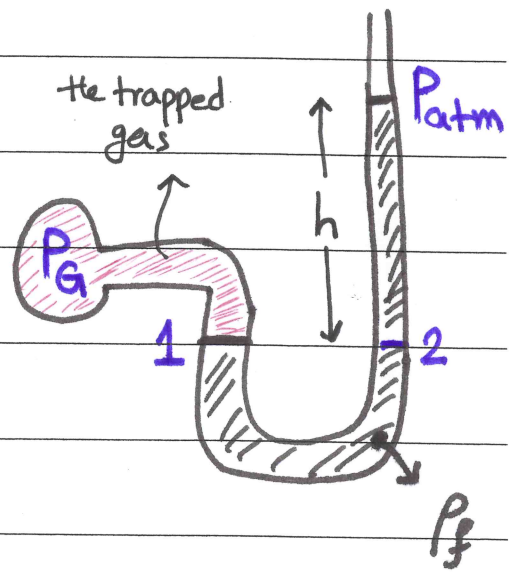
so $76\ \text{cm-Hg} = 10.3\ \text{m-}H_2O = 1\ \text{atm}$ — (11)

You better ponder the conversion factors among the various units of pressure given in Table 10.2. The table is a "nuisance", Giancoli said!

- Thus, water can not be made to rise more than about 10 m under normal atmospheric pressure (1 atm ⇒ 10.3 m). To pump water (from a well) or to push a fluid through a pipe up to a higher altitude, a force pump is needed. (Check chapter 6: 1 hp water pump - problem 58).
- Problem 10.12: How high would the level be in an alcohol barometer at 1 atm? (13 m) ... check Table 10.1.
- Quiz: For which of the following fluids will the column of fluid in the barometer be the highest?

(a) blood (plasma) (b) sea water (c) gasoline.

- An open-tube manometer measures the gauge pressure, P_G, of a gas. It consists of a U-shaped tube partially filled with a fluid. The closed end of the manometer is connected to a vessel containing



the gas whose gauge pressure, P_G, is being measured, and the other end is open and thus experiences atmospheric pressure, P_{atm}.

Now, $P_1 = P_2$, and eq (5) reads:-

$P_2 = P_{atm} + \rho_f g h$.

Therefore, P_1 (the absolute pressure of the gas trapped in the vessel) is:

$P_1 = P_{atm} + \rho_f g h$.

The gauge pressure of the gas in the vessel is then:

$P_G = P_1 - P_{atm} = \rho_f g h$, notice that $\rho_f \equiv \rho_{fluid}$.

- Blood pressure is measured using a gauge that is usually calibrated in mm-Hg (go back to page 9).

The gauge is attached to a closed, air-filled cuff that is wrapped around the upper arm at the level of the heart: Check sec 10.14. Two values of blood pressure are measured: the maximum pressure when the heart is pumping, called systolic pressure; and the pressure

when the heart is in the resting part of the cycle, called diastolic pressure. Normal systolic pressure is around 120 mm-Hg, whereas normal diastolic pressure is around 80 mm-Hg. Blood pressure is reported in the form 120/80.

- Quiz: If your systolic pressure is 150 mm-Hg (this is hardly surprising being a PHY 105 @ JU ...!), what is the absolute pressure?
- Please do NOT take the physician mentioned in Problem 10.66 as a role model! You do not need to emulate a physician attaching or wrapping the cuff around a patient's "calf" instead his arm ...!

III Section 10:5 = Pascal's principle:

- Recall eq(8): $P_{absolute} = P_{atm} + \rho g h$.

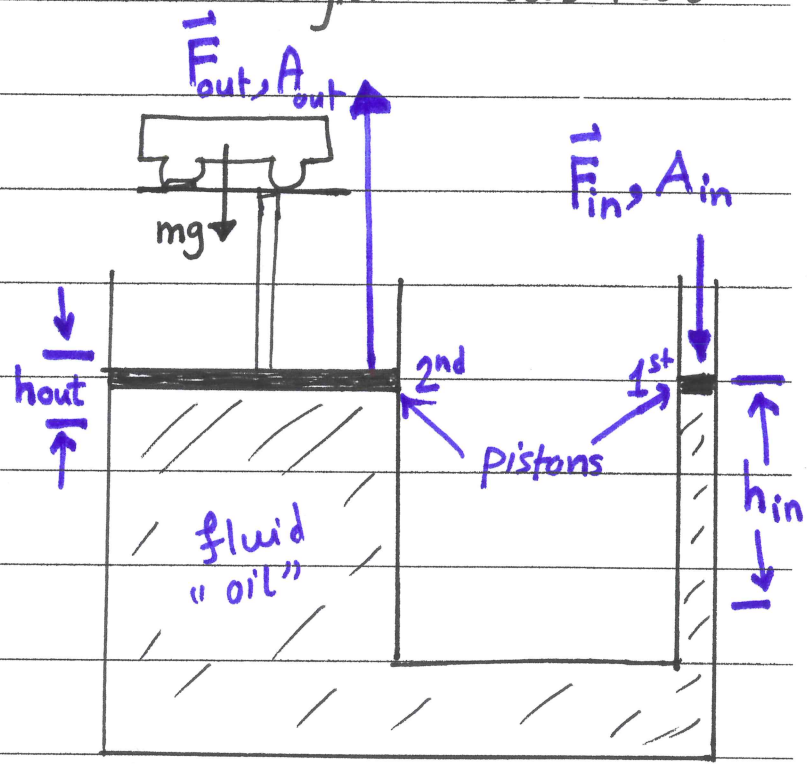
Suppose, now, that P_{atm} is increased from P_{atm} to $P_{atm} + \Delta P$. As a result, the pressure at the depth h is also increased:

$P_{absolute} = P_{atm} + \Delta P + \rho g h = (P_{atm} + \rho g h) + \Delta P$.

Thus, by increasing the pressure at the top of the fluid by the amount ΔP, we have increased the pressure by the same amount everywhere in the fluid. This is Pascal's principle, which can be stated as follows:

"If pressure is exerted on a part of a fluid, that pressure will be transmitted to all parts of the fluid without loss."

- A classic example of Pascal's principle at work is the hydraulic lift. The schematic diagram shows two pistons in container:-



- One piston has area A_{in} and the other piston has area A_{out}, with $A_{in} < A_{out}$.
- A force, F_{in}, is exerted on the 1st piston, producing a change in pressure in the oil.

This change in pressure is transmitted to all points in this oil, including points adjacent to the 2nd piston. We can write:

$$\Delta P = \frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \quad \text{or}$$

$$F_{out} = F_{in}\left[\frac{A_{out}}{A_{in}}\right] \quad \text{——— (12)}$$

Because A_{out} is larger than A_{in}, F_{out} is larger than F_{in}. Thus, the force applied to the 1st piston is magnified. This phenomenon is the basis of hydraulic devices that produce large output forces with small input forces: $[F_{out} = mg]$.

- Recall the mechanical advantage with a lever in Example 9.3
- The amount of work done on the 1st piston is the same as the amount of work done by the 2nd piston. To calculate the work done, we need to calculate the distance over which the forces act. For both pistons, the volume, V, of oil that is moved is the same:

$$V = h_{in} A_{in} = h_{out} A_{out} \quad ; \text{"no leakage} \Rightarrow \text{confined"}$$

where h_{in} is the distance the 1st piston moves and h_{out}

is the distance the 2nd piston moves. We can see that

$$h_{out} = h_{in}\left[\frac{A_{in}}{A_{out}}\right] \quad \text{——— (13)}$$

which means that the 2nd piston moves a smaller distance than the 1st piston because $A_{in} < A_{out}$. We can find the work done by using the fact that work is force times distance and using eqs (12) and (13):

$$W = F_{in}\, h_{in} = \left[F_{out}\,\frac{A_{in}}{A_{out}}\right]\left[h_{out}\,\frac{A_{out}}{A_{in}}\right] = F_{out}\, h_{out} \quad \text{— (14)}$$

Thus, this hydraulic device transmits a larger force over a smaller distance. However, no additional work is done.

- To be specific, let's assume that $A_{out} = 100\, A_{in}$. Then, eq (12) reads that $F_{out} = 100\, F_{in}$. Our force (F_{in}) has been magnified 100 times! If this sounds too good to be true, rest assured that we are not getting something for nothing. Just as with a lever (ex 9.3), there is a tradeoff between the distance through which a force

must be applied and the force magnification.
Thus, in the example just given, eq (13) reads that $h_{out} = h_{in}/100$. Our force at the 2nd piston has been magnified 100 times, but the distance it moves has been reduced 100 times. This is what eq (14) tells us about!

IV Section 10:7

Buoyant force and Archimedes' principle.

- The upward force exerted by a fluid on any fully or partially submerged (immersed) object is called a buoyant force. The buoyant force is what allows huge ships made of steel to float on the surface of the ocean. It is extremely difficult to push a beach ball down under water because of the large upward force exerted by the water on the ball.
- Whether you notice it or not, we have determined buoyant force on pages 4 and 5!

The figure in page 4 shows a cube of fluid within a larger volume of the fluid. The weight of the cube is supported by the force resulting from the pressure difference between the top and bottom surfaces of the cube, as given by eq (4)

$F_2 - F_1 = mg$, where m is the mass of the fluid in the cube.

∴ Archimedes' principle states that when an object is fully or partially submerged in a fluid, the fluid pushes upward with a buoyant force with magnitude

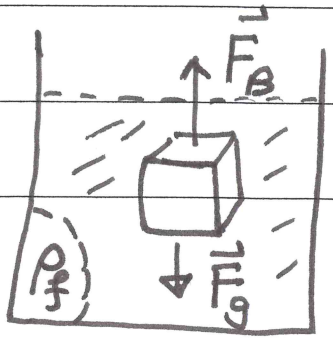
$$F_B = m_f g \quad \text{——— (15)}$$

, where m_f is the mass of the fluid in the cube:

$$m_f = V_f \rho_f \quad \text{——— (16)}$$ (page 5).

- Now suppose the cube of fluid is replaced with a cube of solid material.

The external forces acting on the submerged cube are



the gravitational force $\vec{F}_g$ and the buoyant force $\vec{F}_B$.

$\vec{F}_g = m_o \vec{g}$, m_o = mass of the object itself. $= V_o \rho_o$.

$\vec{F}_B = m_f \vec{g}$, m_f = mass of the fluid $= V_f \rho_f$.

Thus, the net force acting on the cube is:

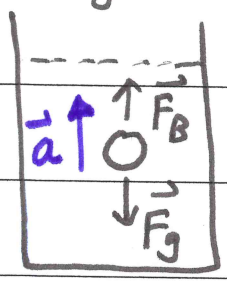
$$\vec{F}_{net} = F_B - F_g = g(V_f \rho_f - V_o \rho_o) \quad \text{— (17)}$$

- Let's discuss two common situations concerning eq 17:

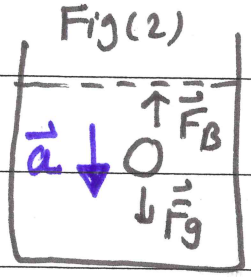
I) Totally submerged object: $[V_f = V_o]$.

As the object is totally submerged, the volume of the fluid displaced by the object (i.e the volume of the fluid that has been pushed out of the way by the object) is equal to the volume of the object itself: $V_f = V_o$.

Fig(1)



Fig(2)



$\therefore$ Eq 17 reads $\vec{F}_{net} = g V_o (\rho_f - \rho_o)$

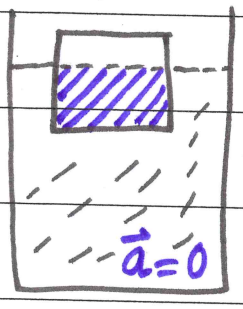
Hence, if $\rho_f > \rho_o$, the submerged object accelerates upward (Fig(1)-page 21). A block of wood held under water and released will rise to the surface.

If $\rho_f < \rho_o$, the submerged object accelerates downward. A rock will sink to the bottom when released in water. (Fig(2)- page 21). If $\rho_f = \rho_o$, $\vec{F}_{net} = 0$, and the block remains in equilibrium. Therefore, the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid!

II) Floating (partly submerged) object: $[V_f < V_o]$.

Consider the object (V_o and ρ_o) with $\rho_f > \rho_o$ in Fig 1 - page 21 after it reaches the surface. After bobbing a bit, it will settle into static equilibrium on the surface of the fluid: it will float, and will be only partially submerged (Fig 3).

Fig(3)

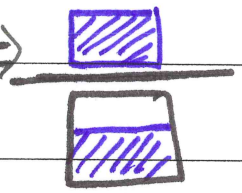


In this case, $\vec{F}_{net}$ (as given by eq 17) $= 0$.
We no longer have $V_f = V_0$ as in case I. This is because only a portion of the object's volume is below the surface of the fluid. Thus, V_f (the volume displaced) is the same as the volume of that part of the object beneath the surface of the fluid: the blue part in Fig (3).

$$\Rightarrow \vec{F}_{net}\ (\text{eq } 17) = 0 \Rightarrow V_f \rho_f = V_0 \rho_0$$

$$\Rightarrow \frac{V_f}{V_0} \equiv \frac{V_{displaced}}{V_0} = \frac{\rho_0}{\rho_f} \quad \text{——— (18)}$$

$$\therefore \frac{V_f}{V_0} < 1 \Longrightarrow$$



$< 1.$

- We conclude this section by exploring few examples.

[1] Exercise D in your text: Which of the following objects submerged in water, experiences the largest magnitude of the buoyant force?

$1\,m^3$ helium ball, $1\,m^3$ wood, $1\,m^3$ ice, $1\,m^3$ iron, all the same

Remember that the buoyant force is exerted by the fluid. It is not determined by the properties of the object except for the amount of fluid displaced by the object. Therefore, if several objects of different densities but the same volume are immersed in a fluid, they will all experience the same buoyant force. This is verified by eqs (15)-(16)

$$F_B = m_f g = V_f \rho_f g, \quad V_f = V_0 \Rightarrow F_B = \rho_f g \left[V_0\right].$$

$\boxed{2}$ Exercise C: the same question but for:

1 kg helium balloon, 1 kg wood, 1 kg ice, 1 kg iron, all the same.

$$F_B = \rho_f g \left[V_0 = \frac{m_0}{\rho_0}\right] = \left[\rho_f g m_0\right] \frac{1}{\rho_0} \Rightarrow \therefore \text{helium}.$$

$\boxed{3}$ Tip of the iceberg:

How much of a floating iceberg is submerged in water?

This example corresponds to case II (page 22) because only part of the iceberg is underneath the water.

Evaluate eq (18 using the densities of ice and seawater (Table 10.1):

$$\text{fraction } f = \frac{V_f}{V_0} = \frac{\rho_0}{\rho_f} = \frac{\rho_{ice}}{\rho_{seawater}} = \frac{917\ kg/m^3}{1030\ kg/m^3} = 0.89.$$

Therefore, the visible fraction of iceberg above the water's surface is about 11%.

You are likely familiar with the phrase, "That's only the tip of the iceberg." It is the unseen 89% below the water that represents the danger to a passing ship!

- Do problem 10.23: piece of iron floats in mercury....
- Do problem 10.31: SG of ice = 0.917....

4 Eureka, Archimedes shouted!

According to legend, the king, Hiero of Syracuse, ordered a new crown made and gave the goldsmith the exact amount of gold needed to create the crown. When the crown was finished, it had the correct weight, but Hiero suspected that the goldsmith had used some silver in the crown to replace the more valuable gold. Hiero could not prove this and went to Archimedes for help. Archimedes

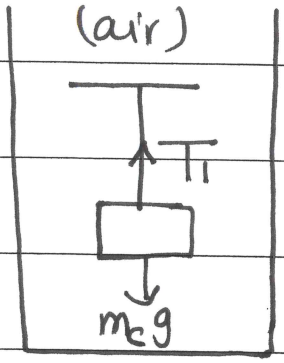
was able to demonstrate the silver-for-gold switch and thus prove the theft committed by the goldsmith. Archimedes needed to measure the density of the crown to find out what fraction of it was gold ($19.3\ g/cm^3$) and what fraction was silver ($10.5\ g/cm^3$). However, how could he measure the density? He needed to submerge the crown in water and determine its volume from the rise in the water level. Weighing the crown gave him the mass, and dividing the mass by the volume yielded the density, the answer Hiero wanted.

- This problem corresponds to case I (page 21) because the crown is completely submerged.

We will solve the problem using newton's law (1643-1726) and the buoyant force, without the need to measure water levels (Archimedes' method: 287-212 BC)!!

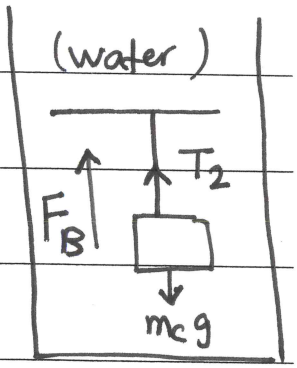
We will solve the problem by weighing the crown first in air and then in water. Owing to the fact that the density of air ($1.29\ kg\,m^{-3}$) is much smaller than the density of water ($10^3\ kg\,m^{-3}$), we will neglect the small buoyant force due to air!

- When the crown is suspended in air, the scale reads the true weight:



$$T_1 = m_c g \quad \text{—— (1)}$$

- When the crown is immersed in water, the buoyant force F_B reduces the scale reading to an apparent weight of: $T_2 = m_c g - F_B$



$$\therefore\ T_2\,[\text{apparent}] = T_1\,[\text{real}] - F_B \quad \text{—— (2)}$$

- Solve for $F_B = T_1 - T_2$. Recall eqs (15) and (16):

$$F_B = m_f g = V_f \rho_f g,\ V_f = V_c \Rightarrow F_B = V_c \rho_f g \quad \text{—— (3)}$$

- Find the density of the crown $\rho_c = \dfrac{m_c}{V_c} = \dfrac{m_c}{[F_B/\rho_f g]}$

$$\therefore\ \rho_c = \frac{[m_c g]\,\rho_f}{F_B}$$

$$\Rightarrow \underset{\text{unknown}}{\rho_c} = \underbrace{\left[\frac{T_1}{T_1 - T_2}\right]\rho_f}_{\text{known}} \quad \text{—— (4)}$$

- Suppose the scale read 7.84 N when the crown was in air and 6.84N when it was in water. What should Archimedes have told Hiero?

Substitute numerical values in (4)

$$\rho_c = \left[\frac{7.84}{7.84-6.84}\right]10^3 = 7.84\,10^3\ kg/m^3$$

From Table 10.1, $\rho_{gold} = 19.3\,10^3\ kg/m^3$. Therefore, Archimedes should have reported that Hiero had been cheated by the goldsmith!

- Suppose the crown has the same weight but is indeed pure gold. What would the scale reading be when the crown is immersed in water?

Find F_B (3) $= \rho_f g V_c = \rho_w g\left(\frac{m_c}{\rho_c}\right) = \rho_w\left(\frac{m_c g}{\rho_c}\right)$

$$\therefore F_B = \frac{10^3 * T_1}{19.3\,10^3} = 0.406\,N$$

$$T_2\ (2) = T_1 - F_B = 7.43\,N$$

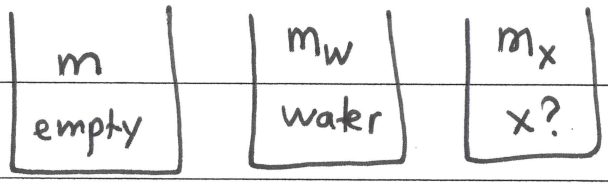
- You have noticed that the gold crown (with higher density than water) has experienced an upward buoyant force that is less than its real weight. Its apparent weight is then given by

$$\left[\text{apparent weight} = \text{actual weight} - \text{buoyant force}\right] \quad (19)$$

Eq (19) is exactly eq (2) in our example.

- Do examples 10.7 and 10.8.
- Do problem 10.25: a crane lifts 1810^3 kg steel...
- Do problem 10.27: mass of 63.5g → air, mass of 55.4g, water...

[5] Problem 10.5:



SG_x ?

notice that $m_x < m_w$.

$$\Rightarrow SG_x \Rightarrow \text{recall eq (1)} = \frac{\rho_{(x)}}{\rho_{water}} = \frac{(m_x - m)/V}{(m_w - m)/V}$$

$$\therefore SG_x = \frac{m_x - m}{m_w - m}$$, plug in the numerical values.

- Can you anticipate that SG_x is less than 1?

6 Problem 10.6: antifreeze : 4L , SG = 0.8

water : 5L

⇒ add both to make a 9L mixture. ⇒ SG of the mixture?

$$SG_{mixture} = \frac{\rho_{mixture}}{\rho_{water}} = \frac{(m_{anti} + m_W)/V_{mix}}{\rho_W}$$

$$SG_{mixture} = \frac{1}{V_{mix}} \left[\frac{\rho_{anti} * V_{anti} + \rho_W * V_W}{\rho_W} \right]$$

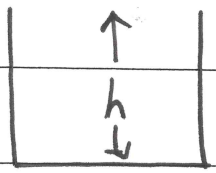
$$= \frac{1}{V_{mix}} \left[SG_{anti} * V_{anti} + V_W \right] = \frac{0.8 * 4 + 5}{9}.$$

7 Problem 10.14: the max gauge pressure in a hydraulic lift is 17 atm, the diameter of the output piston is 25.5 cm. What is the largest possible mass that can be lifted?

$F_{out} = P * A = mg$ ⇒ solve for m:

$m = \left(P * \frac{\pi}{4} d^2\right)/g$. [Look at the schematic diagram - page 16].

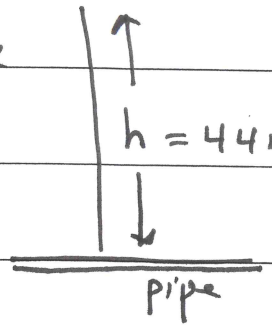
8 Problem 10.16: total force and the absolute pressure on the bottom of the pool?



$= P_{absolute} = P_{atm} + \rho_{water}\, g\, h = (\ )$ Pa.

$= F_{total} = P_{absolute} * \underbrace{A_{base}}_{given}$.

[9] Problem 10.20: the minimum gauge pressure needed to support a column of water 40 m high?



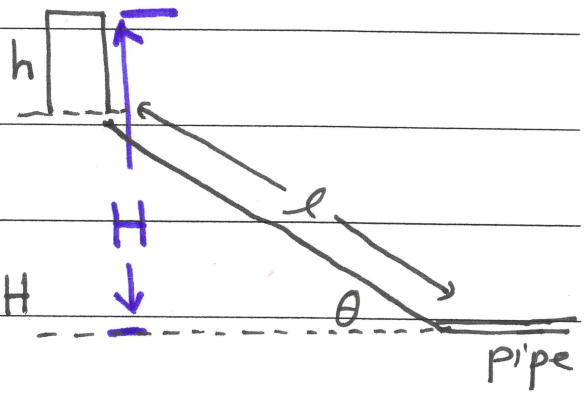
$P_{gauge} = \rho_f g h = 10^3 \times 9.8 \times 44 = 4.3\,10^5$ Pa

$\therefore$ $P_{gauge} = 4.24$ atm.

Or $\Rightarrow$ recall eq (11) 10.3 m-H_2O $\longrightarrow$ 1 atm

44 m-H_2O $\longrightarrow$? atm

[10] Problem 10.17:



(a) Given h, l and θ, determine the water gauge pressure in the pipe.

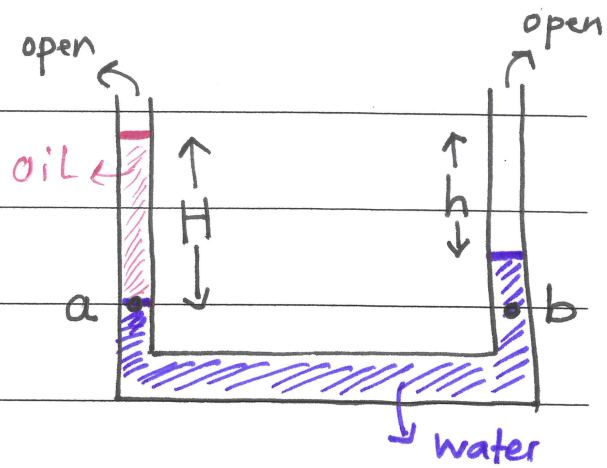
$P_{gauge} = P_{absolute} - P_{atm} = \rho_{water} * g * H$

$H = l \sin\theta + h$.

(b) how high could the water shoot if it came vertically out of a broken pipe? This is answered properly by Bernoulli's equation [eq 10.5 in your text], yet intuitively and neglecting air resistance, it will reach up to H.

[11] Problem 10.43: What gauge pressure in water pipe to shoot water to h = 16 m? $\Rightarrow$ intuitively $= \rho_{water} * g * 16$ ✓

Cheer up! you can solve it without consulting Bernoulli's eq in section 10.9.



12 Problem 10.18:

Both water and oil are at rest.

Given H and h, what is ρ_{oil}?

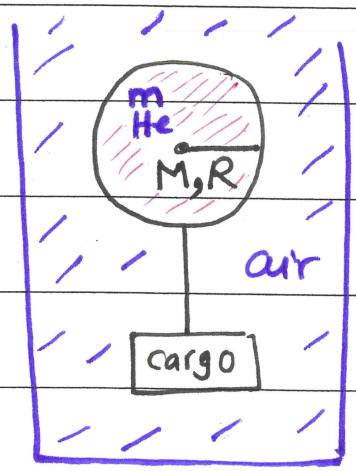
Recall page 11 when we argued why P_a must equal P_b.

$\Rightarrow P_a = P_b$ [$P(1) = P(1)$ in page 11]

$P_{atm} + \rho_{oil} g H = P_{atm} + \rho_{water} g (H-h)$ → solve for ρ_{oil}

$$\rho_{oil} = \rho_{water} * \left[\frac{H-h}{H}\right].$$

- Can you anticipate why $\rho_{oil} < \rho_{water}$?



13 Problem 10.26: Given a spherical balloon of mass M and radius R. The balloon is filled with helium (m_{He}). What is the largest cargo (m) this balloon can lift? Ignore F_B on cargo.

$F_{B\,air} = (m + M + m_{He}) g$

$\rho_{air} V_{air} g = (m + M + m_{He}) g \Rightarrow V_{air} = V_{balloon} \Rightarrow$ solve for m:

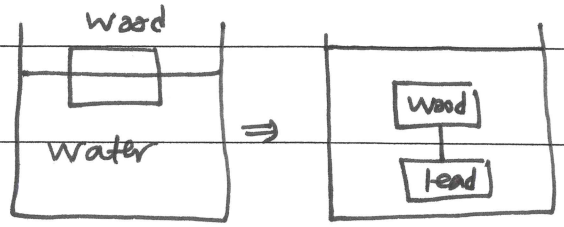
$m = \rho_{air} V_{balloon} - M - m_{He}$, $m_{He} = \rho_{He} * V_{balloon}$

$$\therefore \quad m = \underbrace{V_{balloon}}_{\frac{4}{3}\pi R^3}\left[\underbrace{\rho_{air}}_{1.29} - \underbrace{\rho_{He}}_{0.179}\right] - \underbrace{M}_{930}.$$

- Do example 10.10

14) Problem 10.36: A 3.65 kg block of wood (SG = 0.5) floats on water. What minimum mass of lead (recall page 2: SG = 11.3), hung from the wood by a string, will cause the block to sink?

⇒ minimum mass of Pb means F_{net} (eq 17) = 0 i.e the wood will sink to the bottom without acceleration: sinks with constant speed; the "terminal speed".



$$\therefore F_{B_{water}} = (m_{wood} + m_{Pb})\, g$$

$$\rho_{water} V_{water}\, g = (m_{wood} + m_{Pb})\, g \quad , \quad V_{water} \equiv V_{displaced} = V_{wood} + V_{Pb}$$

$$\therefore \rho_{water}\left[V_{wood} + V_{Pb}\right] = m_{wood} + m_{Pb}$$

$$\rho_{water}\left[\frac{m_{wood}}{\rho_{wood}} + \frac{m_{Pb}}{\rho_{Pb}}\right] = m_{wood} + m_{Pb}$$

V_{air} in problem 13 equal $V_{balloon}$ after neglecting V_{cargo}

recall eq (1) $SG(x) = \rho(x)/\rho_{water}$ ⇒

$$\frac{m_{wood}}{SG_{wood}} + \frac{m_{Pb}}{SG_{Pb}} = m_{wood} + m_{Pb} \text{ , solve for } m_{Pb}$$

$$m_{Pb} = m_{wood}\left[\left(\frac{1 - SG_{wood}}{SG_{Pb} - 1}\right)\left(\frac{SG_{Pb}}{SG_{wood}}\right)\right]$$