Parametric versus Nonparametric (Chi-square)

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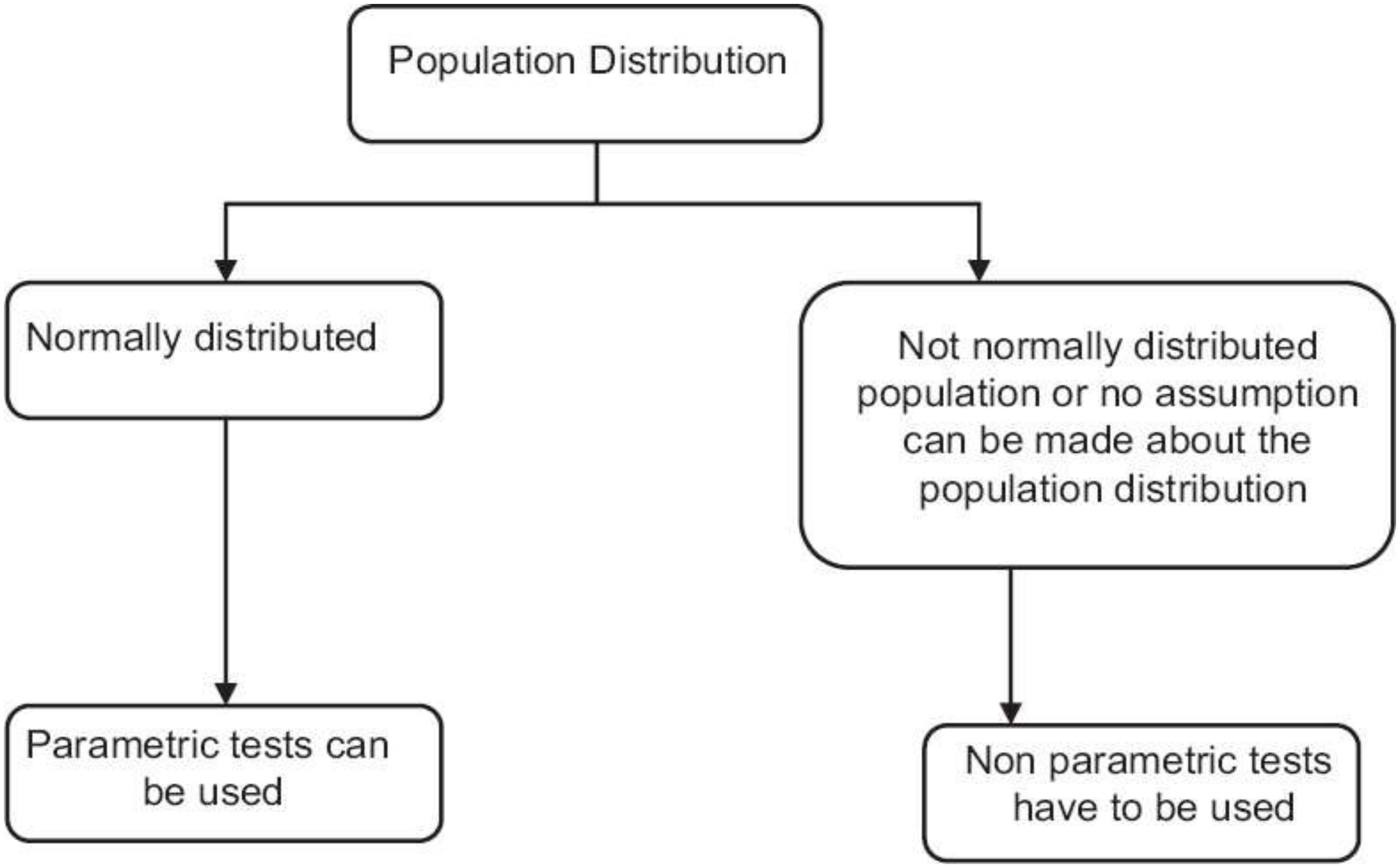

Population Distribution
Normally distributed
Not normally distributed population or no assumption can be made about the population distribution
Parametric tests can be used
Non parametric tests have to be used

Parametric Assumptions

✓The observations must be independent.

✓Dependent variable should be continuous (I/R)

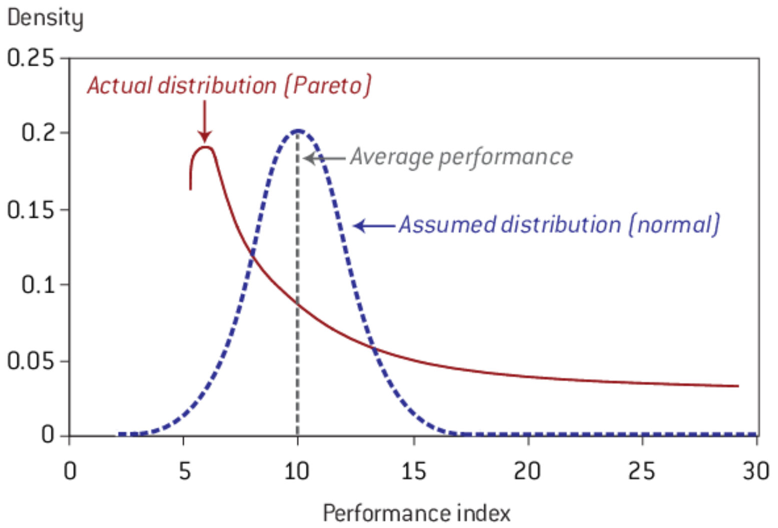
✓The observations must be drawn from normally distributed populations

✓These populations must have the same variances. Equal variance (homogeneity of variance)

✓The groups should be randomly drawn from normally distributed and independent populations

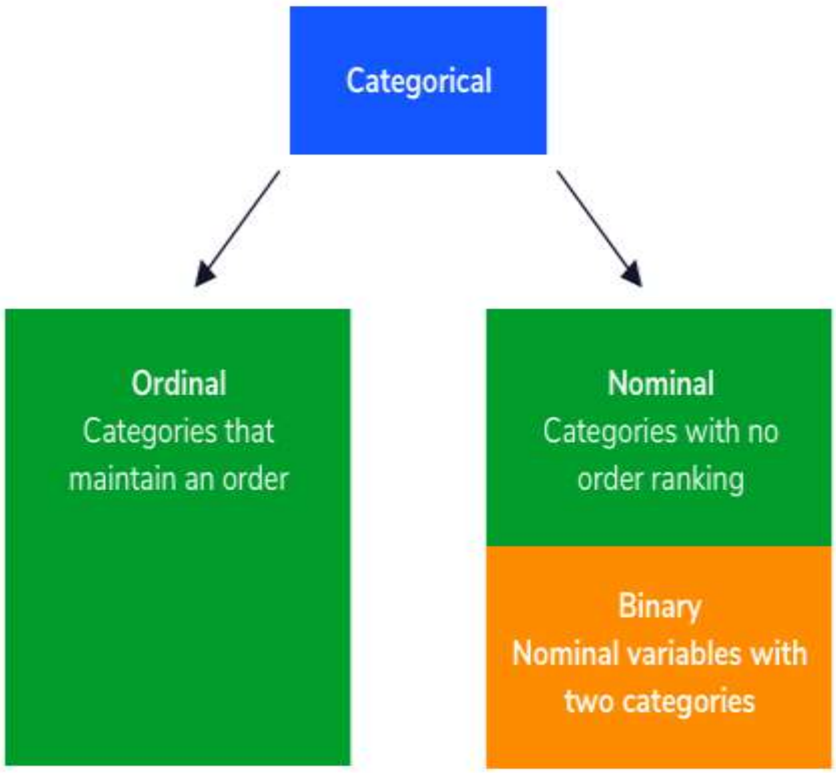
e.g.
Male X Female
Pharmacist X Physician
Manager X Staff

✓**NO OVER LAP**



Parametric Assumptions

- The independent variable is categorical with two or more levels.
- Distribution for the **two or more independent** variables is normal.



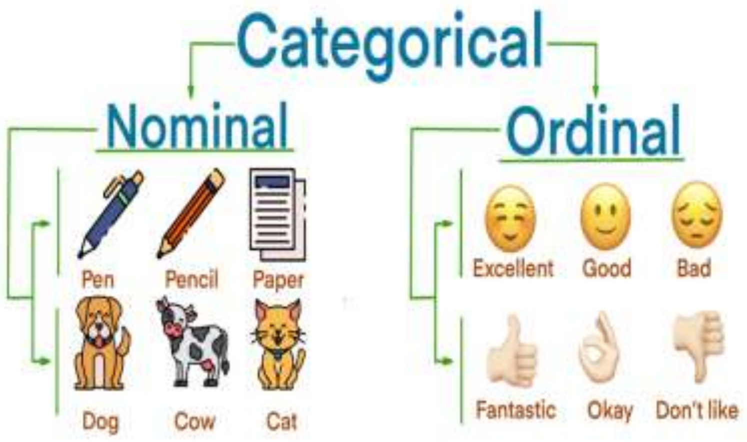
Advantages of Parametric Techniques

They are more powerful and more flexible than nonparametric techniques.

They not only allow the researcher to study the effect of many independent variables on the dependent variable, but they also make possible the study of their interaction.

Nonparametric Methods

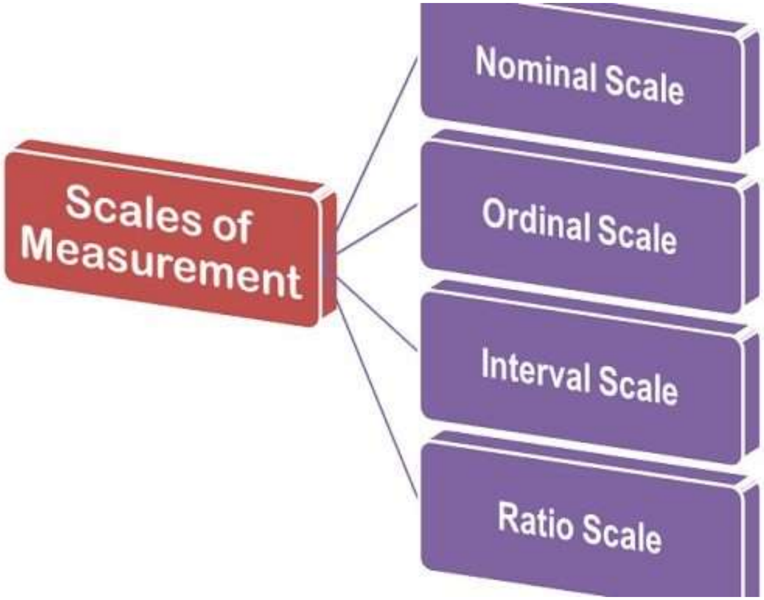
- Nonparametric methods are often the only way to analyze nominal or ordinal data and draw statistical conclusions.
- Nonparametric methods require no assumptions about the population probability distributions.
- Nonparametric methods are often called distribution-free methods.
- Nonparametric methods can be used with small samples



Nonparametric Methods

In general, for a statistical method to be classified as nonparametric, it must satisfy at least one of the following conditions.

- The method can be used with nominal data.
- The method can be used with ordinal data.
- The method can be used with interval or ratio data when no assumption can be made about the population probability distribution (in small samples).



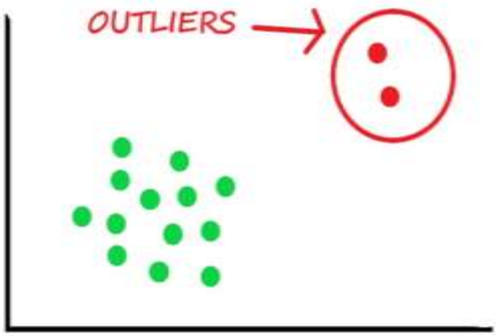
Non Parametric Tests

Do not make as many assumptions about the distribution of the data as the parametric (such as *t* test)

- Do not require data to be Normal
- Good for data with outliers

Non-parametric tests based on ranks of the data

- Work well for ordinal data (data that have a defined order, but for which averages may not make sense).

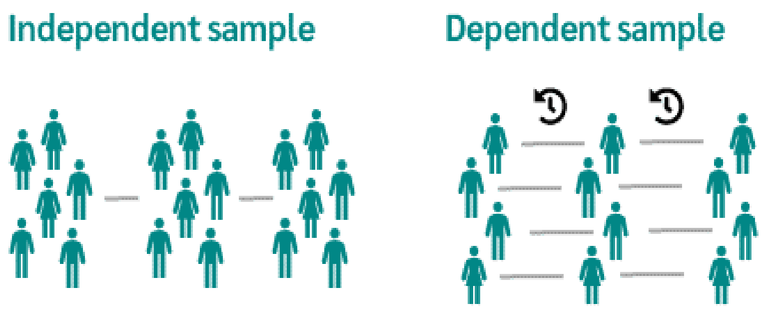


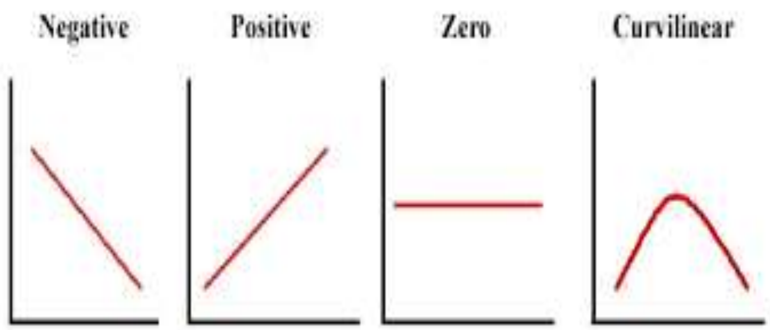
Nonparametric Methods

There is at least one nonparametric test equivalent to each parametric test

These tests fall into several categories

1. Tests of differences between groups (independent samples)
2. Tests of differences between variables (dependent samples)
3. Tests of relationships between variables





Summary Table of Statistical Tests

Level of Measurement	Sample Characteristics					Correlation
	1 Sample	2 Sample		K Sample (i.e., >2)		
		Independent	Dependent	Independent	Dependent	
Categorical or Nominal	X^2	X^2	Macnarmar's X^2	X^2	Cochran's Q	
Rank or Ordinal		Mann Whitney U	Wilcoxin Matched Pairs Signed Ranks	Kruskal Wallis H	Friendman's ANOVA	Spearman's rho
Parametric (Interval & Ratio)	z test or t test	t test between groups	t test within groups	1 way ANOVA between groups	1 way ANOVA (within or repeated measure)	Pearson's r
		Factorial (2 way) ANOVA				

Summary: Parametric vs. Nonparametric Statistics

Parametric Statistics are statistical techniques based on assumptions about the population from which the sample data are collected.

- Assumption that data being analyzed are randomly selected from a normally distributed population.
- Requires quantitative measurement that yield interval or ratio level data.

Nonparametric Statistics are based on fewer assumptions about the population and the parameters.

- Sometimes called "distribution-free" statistics.
- A variety of nonparametric statistics are available for use with nominal or ordinal data.

Chi-Square

Types of Statistical Tests

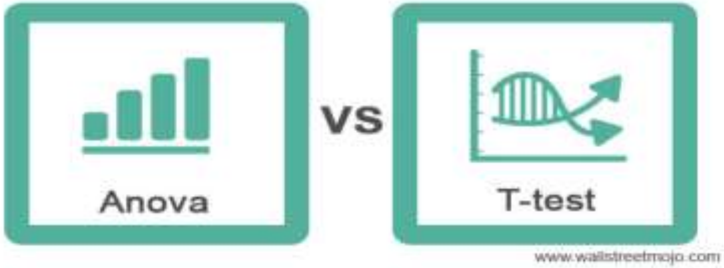
When running a *t test* and *ANOVA*

We compare:

- Mean differences between groups

We assume

- random sampling
- the groups are homogeneous
- distribution is normal
- samples are large enough to represent population (>30)
- DV Data: represented on an **interval or ratio** scale



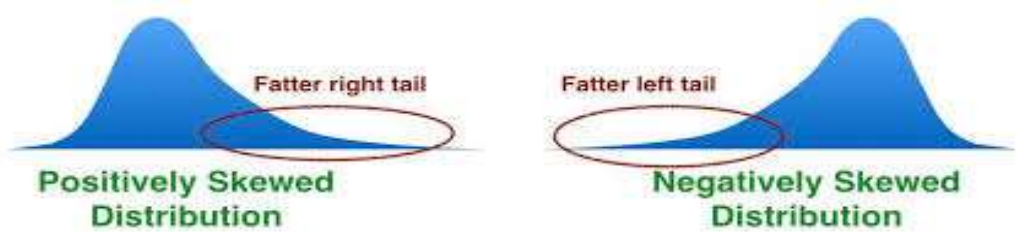
These are Parametric tests!

Types of Tests

When the assumptions are violated:

Subjects were not randomly sampled

DV Data:



- Ordinal (ranked)
- Nominal (categorized: types of car, levels of education, learning styles)
- The scores are greatly skewed or we have no knowledge of the distribution

We use tests that are equivalent to t test and ANOVA

Non-Parametric Test!

Chi-Square test

Must be a random sample from population

Data must be in **raw frequencies**

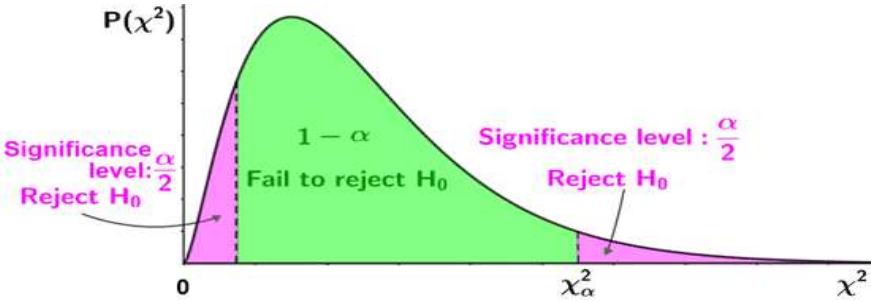
Variables must be independent

A sufficiently large sample size is required (at least 20)

Actual count data (not percentages)

Observations must be independent.

Does not prove causality.



Count of Sex		Health			
Age	Sex	Average	Good	Poor	Grand Total
0-15	Female	8	7	4	19
0-15	Male	5	9	3	17
16-29	Female	9	10	5	24
16-29	Male	4	5	10	19
30-44	Female	2	5	2	9
30-44	Male	5	4	5	14
45-64	Female	4	6	7	17
45-64	Male	4	8	4	16
65+	Female	10	5	10	25
65+	Male	3	8	6	17

Different Scales, Different Measures of Association

Scale of Both Variables	Measures of Association
Nominal Scale	Pearson Chi-Square: χ^2
Ordinal Scale	Spearman’s rho
Interval or Ratio Scale	Pearson r

Important

The chi square test can only be used on data that has the following characteristics:

The data must be in the form of frequencies

The frequency data must have a precise numerical value and must be organised into categories or groups.

The expected frequency in any one cell of the table must be greater than 5.

The total number of observations must be greater than 20.

Degree	Frequency	Relative Frequency	Percentage
High School	2	0.050	5.0
Bachelor's	7	0.175	17.5
MBA	20	0.500	50.0
Master's	3	0.075	7.5
Law	4	0.100	10.0
PhD	4	0.100	10.0
	40		

Formula

$$x^2 = \frac{\sum (O - E)^2}{E}$$

x^2 = The value of chi square
O = The observed value
E = The expected value
$\sum (O - E)^2$ = all the values of (O – E) squared then added together

Chi Square Test of Independence

Purpose

- **To determine if two variables of interest independent (not related) or are related (dependent)?**

- When the variables are independent, we are saying that knowledge of one gives us no information about the other variable. When they are dependent, we are saying that knowledge of one variable is helpful in predicting the value of the other variable.

results of the Chi-square test

The Chi-square independence test showed that gender proportions differed significantly between different levels of sunscreen use (χ^2 = *12.3; df = 2; p = 0.02*), which means that males and females showed different behavior regarding the application of sun protection.

Interpretation of the Chi-square test results

Chi Square Test of Independence

- Some examples where one might use the chi-squared test of independence are:
  - Is level of education related to level of income?
  - Is the level of price related to the level of quality in production?

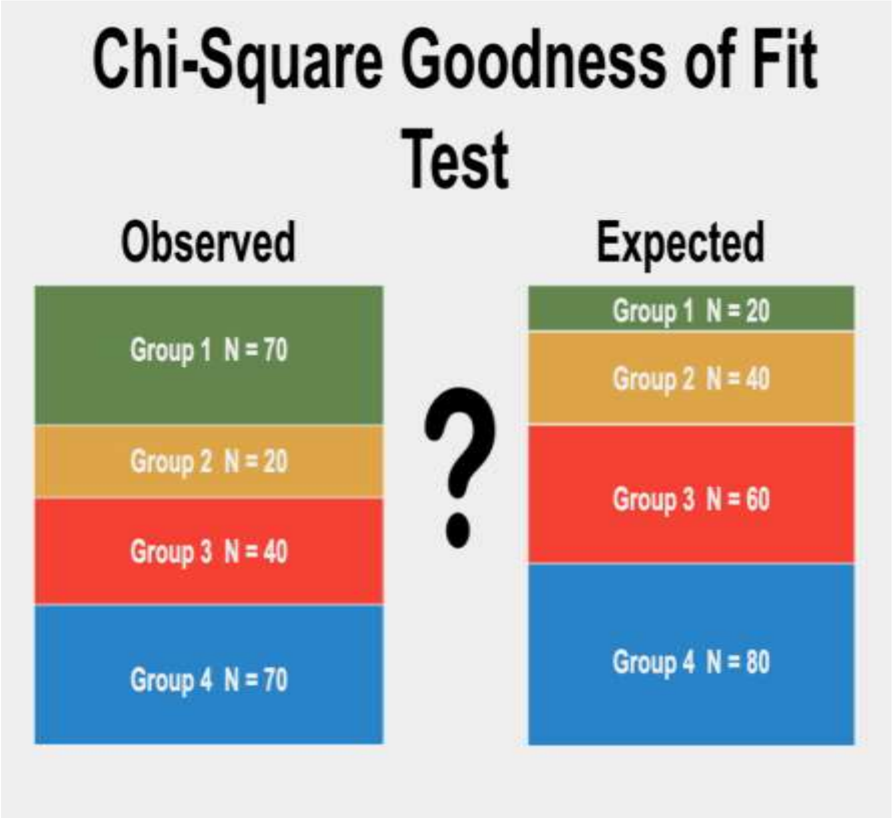
Hypotheses

- The null hypothesis is that the two variables are independent. This will be true if the observed counts in the sample are similar to the expected counts.
  - **H_0: X and Y are independent**
  - **H_1: X and Y are dependent**

Chi Square Test of Goodness of Fit

Purpose

- **To determine whether an observed frequency distribution departs significantly from a hypothesized frequency distribution.**
- **This test is sometimes called a One-sample Chi Square Test.**



Chi Square Test of Goodness of Fit

Hypotheses

- The null hypothesis is that the two variables are independent. This will be true if the observed counts in the sample are similar to the expected counts.
  - **H_0: X follows the hypothesized distribution**
  - **H_1: X deviates from the hypothesized distribution**

Steps in Test of Hypothesis

1. Determine the appropriate test
2. Establish the level of significance: α
3. Formulate the statistical hypothesis
4. Calculate the test statistic
5. Determine the degree of freedom
6. Compare computed test statistic against a tabled/critical value

1. Determine Appropriate Test

Chi Square is used when both variables are measured on a nominal scale.

It can be applied to interval or ratio data that have been categorized into a small number of groups.

It assumes that the observations are randomly sampled from the population.

All observations are independent (an individual can appear only once in a table and there are no overlapping categories).

It does not make any assumptions about the shape of the distribution nor about the homogeneity of variances.

2. Establish Level of Significance

α is a predetermined value

The convention

- $\alpha = .05$
- $\alpha = .01$
- $\alpha = .001$

3. Determine The Hypothesis: Whether There is an Association or Not

H_o : The two variables are independent

H_a : The two variables are associated

4. Calculating Test Statistics

Contrasts observed frequencies in each cell of a contingency table with expected frequencies.

The expected frequencies represent the number of cases that would be found in each cell if the null hypothesis were true (i.e. the nominal variables are unrelated).

Expected frequency of two unrelated events is product of the row and column frequency divided by number of cases.

$$F_e = F_r F_c / N$$

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{Grand total}}$$

4. Calculating Test Statistics

Continued

$$\chi^2 = \sum \left[\frac{(F_o - F_e)^2}{F_e} \right]$$

4. Calculating Test Statistics

Continued

Observed frequencies

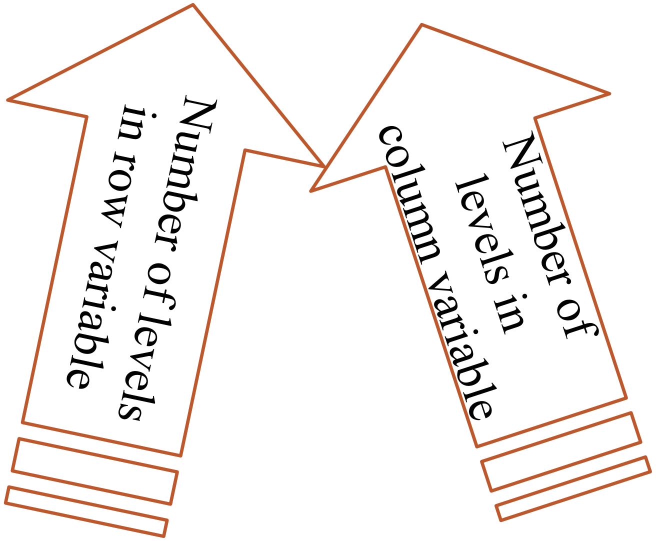
$$\chi^2 = \sum \left[\frac{(F_o - F_e)^2}{F_e} \right]$$

Expected frequency

Expected frequency

5. Determine Degrees of Freedom

df = (R-1)(C-1)



6. Compare computed test statistic against a tabled/critical value

The computed value of the Pearson chi- square statistic is compared with the critical value to determine if the computed value is *improbable*

The critical tabled values are based on sampling distributions of the Pearson chi-square statistic

If calculated χ^2 is greater than χ^2 table value, reject H_o

χ^2

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21

Decision and Interpretation

If the probability of the test statistic is less than or equal to the probability of the alpha error rate, we reject the null hypothesis and conclude that our data supports the research hypothesis. We conclude that there is a relationship between the variables.

If the probability of the test statistic is greater than the probability of the alpha error rate, we fail to reject the null hypothesis. We conclude that there is no relationship between the variables, i.e. they are independent.

Example

Suppose a researcher is interested in voting preferences on gun control issues.

A questionnaire was developed and sent to a random sample of 90 voters.

The researcher also collects information about the political party membership of the sample of 90 respondents.

Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	f_{row}
Democrat	10	10	30	50
Republican	15	15	10	40
f_{column}	25	25	40	n = 90

Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	f row
Democrat	10	10	30	50
Republican	15	15	10	40
f [illegible]n	25	25	40	n = 90

Observed frequencies

Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	f_{row}
Democrat	10	10	30	50
Republican	15	15	10	40
f_{column}	25	25	40	n = 90

Row frequency

Bivariate Frequency Table or Contingency Table

	Favor	Neutral	Oppose	f_{row}
Democrat	10	10	30	50
Republican	15	15	10	40
f n	25	25	40	n = 90

Column frequency

1. Determine Appropriate Test

1. Party Membership (2 levels) and Nominal
2. Voting Preference (3 levels) and Nominal

2. Establish Level of Significance

Alpha of .05

3. Determine The Hypothesis

- Ho : There is no difference between D & R in their opinion on gun control issue.

- Ha : There is an association between responses to the gun control survey and the party membership in the population.

4. Calculating Test Statistics **Continued**

	Favor	Neutral	Oppose	f_{row}
Democrat	f_o =10 f_e =**13.9**	f_o =10 f_e =13.9	f_o =30 f_e=22.2	50
Republican	f_o =15 f_e =11.1	f_o =15 f_e =11.1	f_o =10 f_e =17.8	40
f_{column}	25	25	40	n = 90

= 50*25/90

4. Calculating Test Statistics **Continued**

	Favor	Neutral	Oppose	f_{row}
Democrat	f_o =10 f_e =13.9	f_o =10 f_e =13.9	f_o =30 f_e=22.2	50
Republican	f_o =15 f_e =**11.1**	f_o [illegible] f_e =11.1	f_o =10 f_e =17.8	40
f_{column}	25	25	40	n = 90

= 40* 25/90

4. Calculating Test Statistics Continued

$$\chi^2 = \frac{(10-13.89)^2}{13.89} + \frac{(10-13.89)^2}{13.89} + \frac{(30-22.2)^2}{22.2} +$$

$$\frac{(15-11.11)^2}{11.11} + \frac{(15-11.11)^2}{11.11} + \frac{(10-17.8)^2}{17.8}$$

$$= 11.03$$

5. Determine Degrees of Freedom

df = (R-1)(C-1) =
(2-1)(3-1) = 2

6. Compare computed test statistic against a tabled/critical value

$\alpha = 0.05$

df = 2

Critical tabled value = 5.991

Test statistic, 11.03, exceeds critical value

Null hypothesis is rejected

Democrats & Republicans differ significantly in their opinions on gun control issues

Example 1: Testing for Proportions

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	[illegible]	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	[illegible]	15.51	17.54	20.09
9	2.088	2.700	3.325	[illegible]	[illegible]	16.92	19.02	21.67
10	2.558	3.247	3.940	[illegible]	[illegible]	[illegible]	20.48	23.21

$$\chi^2_{\alpha=0.05} = 5.991$$

SPSS Output for Gun Control Example

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	11.025[a]	2	.004
Likelihood Ratio	11.365	2	.003
Linear-by-Linear Association	8.722	1	.003
N of Valid Cases	90		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 11.11.

Interpreting Cell Differences in a Chi-square Test - 1

MARITAL MARITAL STATUS * SEX RESPONDENTS SEX Crosstabulation

Count

		SEX RESPONDENTS SEX		
		1 MALE	2 FEMALE	Total
MARITAL MARITAL STATUS	1 MARRIED	149	160	309
	2 WIDOWED	12	49	61
	3 DIVORCED	45	59	104
	4 SEPARATED	7	13	20
	5 NEVER MARRIED	80	94	174
Total		293	375	668

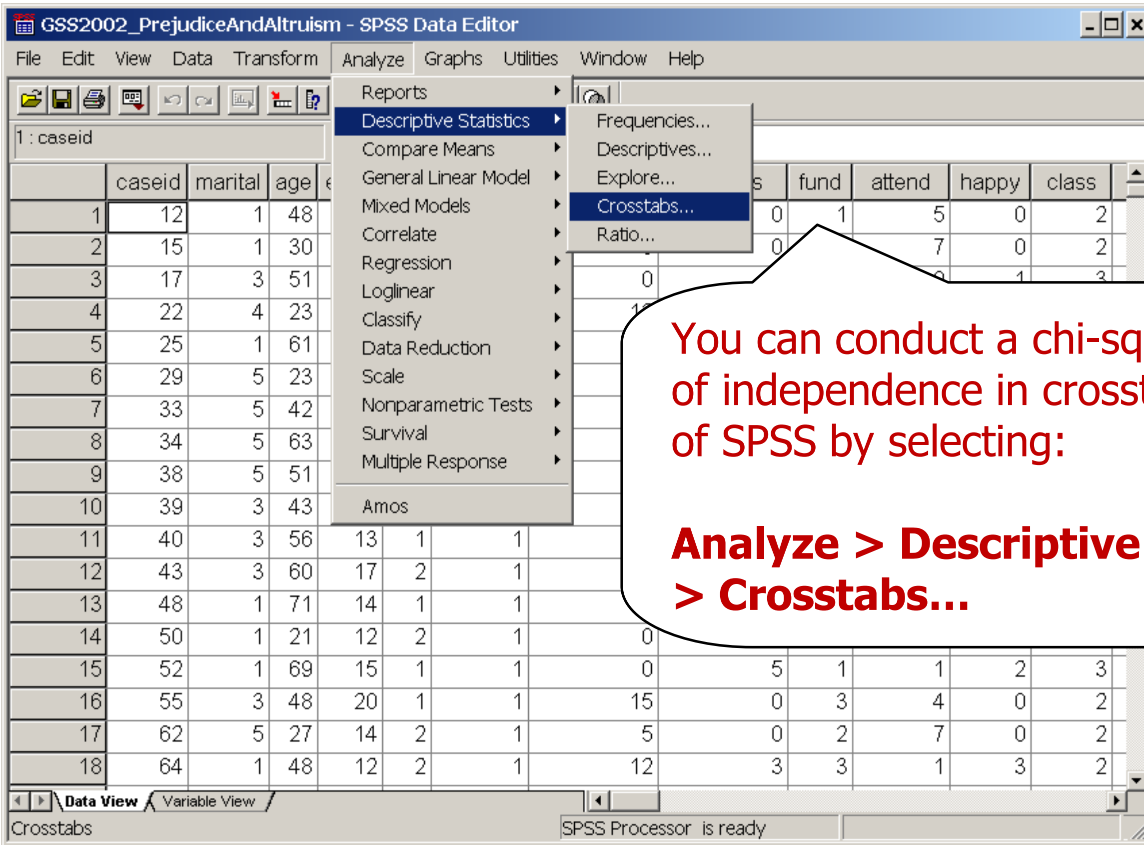
Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	17.848[a]	4	.001
Likelihood Ratio	19.220	4	.001
Linear-by-Linear Association	.094	1	.759
N of Valid Cases	668		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.77.

A chi-square test of independence of the relationship between sex and marital status finds a statistically significant relationship between the variables.

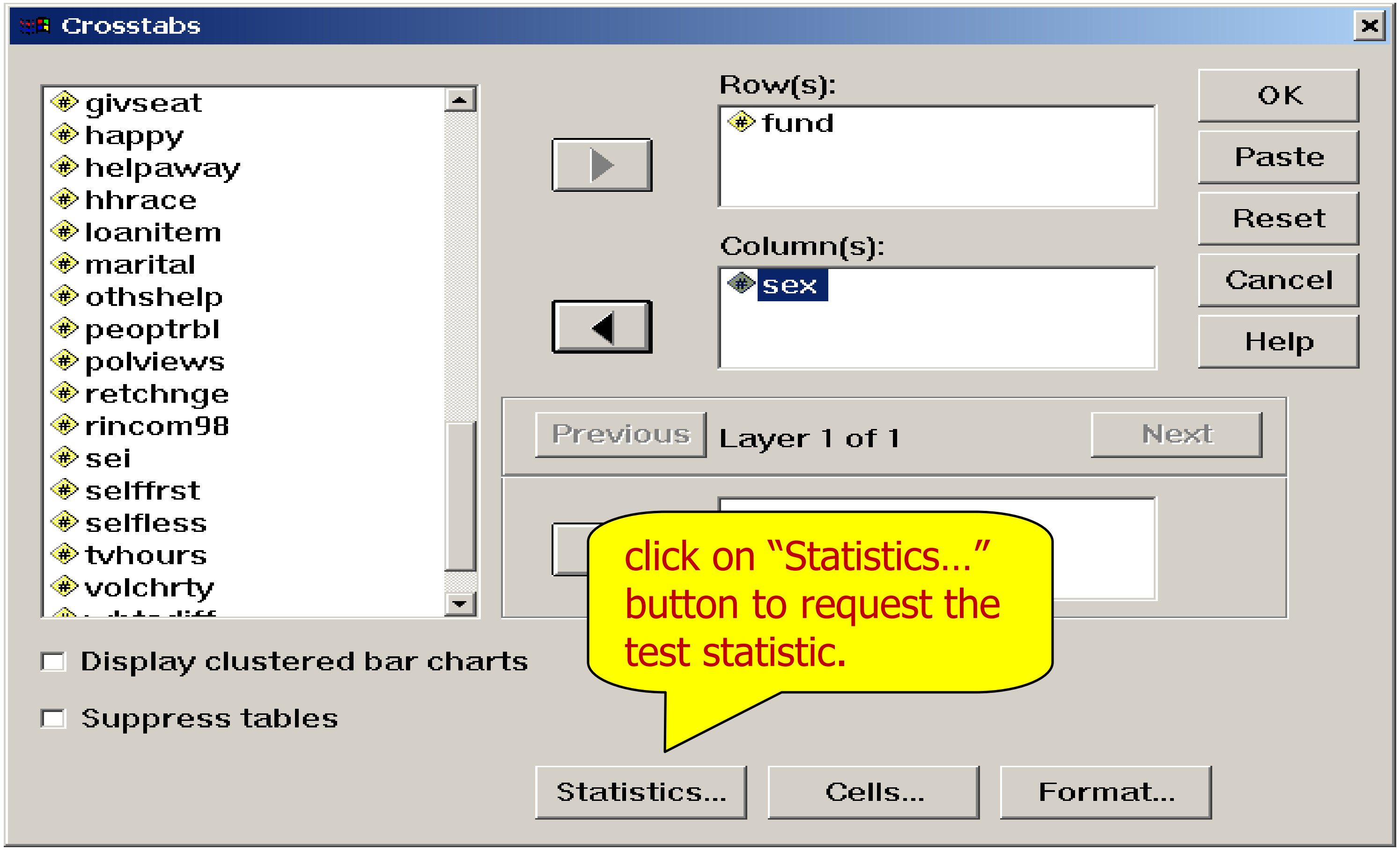
Chi-Square Test of Independence: post hoc test in SPSS (1)



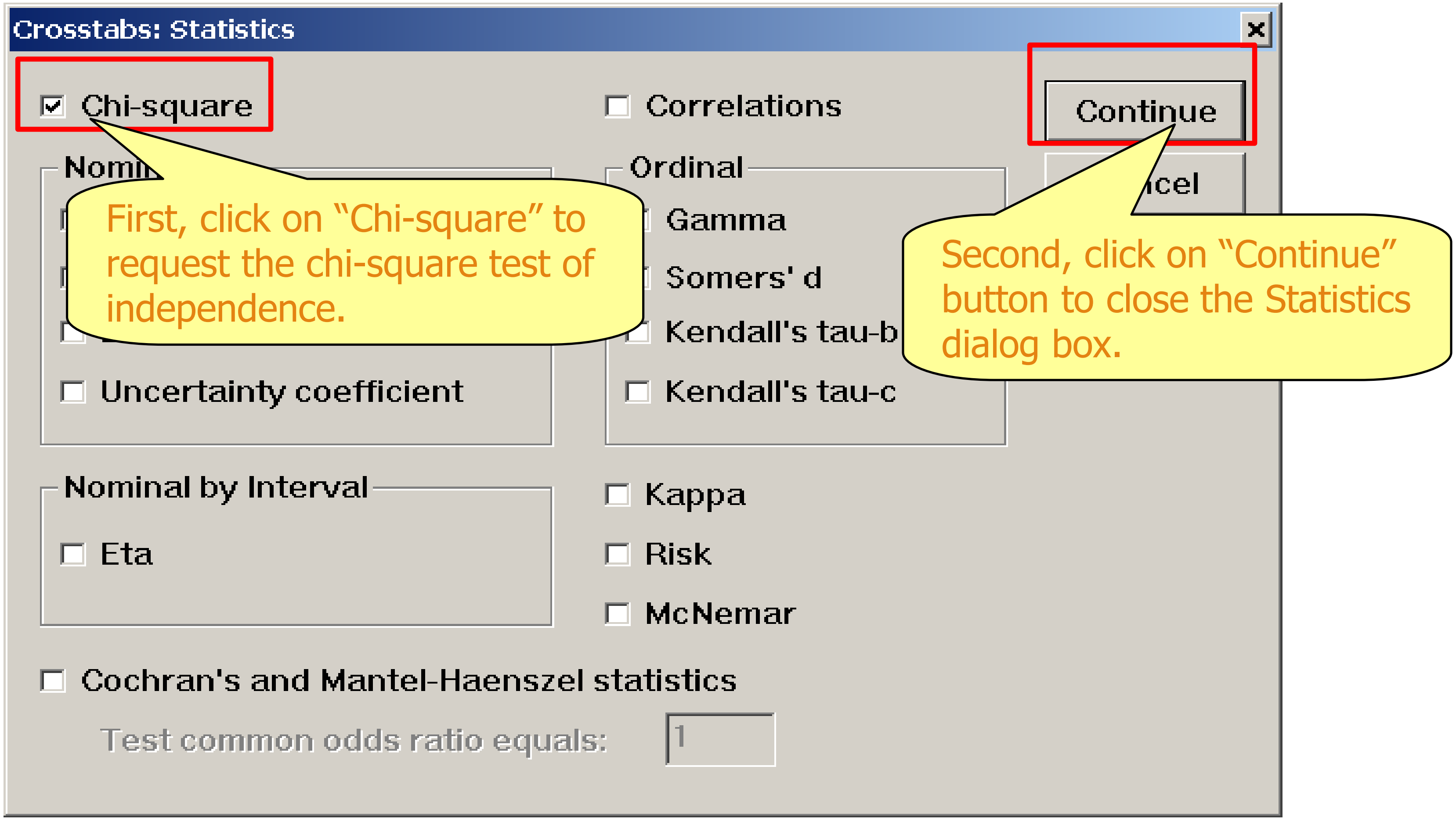
You can conduct a chi-square test of independence in crosstabulation of SPSS by selecting:

Analyze > Descriptive Statistics > Crosstabs…

Chi-Square Test of Independence: post hoc test in SPSS (2)



Chi-Square Test of Independence: post hoc test in SPSS (3)



Chi-Square Test of Independence: post hoc test in SPSS (6)

HOW FUNDAMENTALIST IS R CURRENTLY * RESPONDENTS SEX Crosstabulation

			RESPONDENTS SEX		
			1 MALE	2 FEMALE	Total
HOW FUNDAMENTALIST IS R CURRENTLY	1 FUNDAMENTALIST	Count	75	99	174
		Expected Count	74.9	99.1	174.0
		Residual	.1	-.1	
		Std. Residual	.0	.0	
	2 MODERATE	Count	107	161	268
		Expected Count	115.4	152.6	268.0
		Residual	-8.4	8.4	
		Std. Residual	-.8	.7	
	3 LIBERAL	Count	79	85	164
		Expected Count	70.6	93.4	164.0
		Residual	8.4	-8.4	
		Std. Residual	1.0	-.9	
Total		Count	261	[illegible]	[illegible]
		Expected Count	[illegible]	[illegible]	[illegible]

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	2.821[a]	2	.244
Likelihood Ratio	2.815	2	.245
Linear-by-Linear Association	.832	1	.362
N of Valid Cases	606		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 70.63.

In the table Chi-Square Tests result, SPSS also tells us that “0 cells have expected count less than 5 and the minimum expected count is 70.63”.

The sample size requirement for the chi-square test of independence is satisfied.

Chi-Square Test of Independence: post hoc test in SPSS (7)

HOW FUNDAMENTALIST IS R CURRENTLY * RESPONDENT

HOW FUNDAMENTALIST IS R CURRENTLY	1 FUNDAMENTALIST	Count	
		Expected Count	
		Residual	
		Std. Residual	
	2 MODERATE	Count	
		Expected Count	
		Residual	
		Std. Residual	
	3 LIBERAL	Count	
		Expected Count	
		Residual	
		Std. Residual	
Total		Count	
		Expected Count	

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	2.821[a]	2	.244
Likelihood Ratio	2.815	2	.245
Linear-by-Linear Association	.832	1	.362
N of Valid Cases	606		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 70.63.

The probability of the chi-square test statistic (chi-square=2.821) was p=0.244, greater than the alpha level of significance of 0.05. The null hypothesis that differences in "degree of religious fundamentalism" are independent of differences in "sex" is not rejected.

The research hypothesis that differences in "degree of religious fundamentalism" are related to differences in "sex" is not supported by this analysis.

Thus, the answer for this question is False. We do not interpret cell differences unless the chi-square test statistic supports the research hypothesis.