t-test

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Review 6 Steps for Significance Testing

1. Set alpha (p level).
2. State hypotheses, Null and Alternative.
3. Calculate the test statistic (sample value).
4. Find the critical value of the statistic.
5. State the decision rule.
6. State the conclusion.

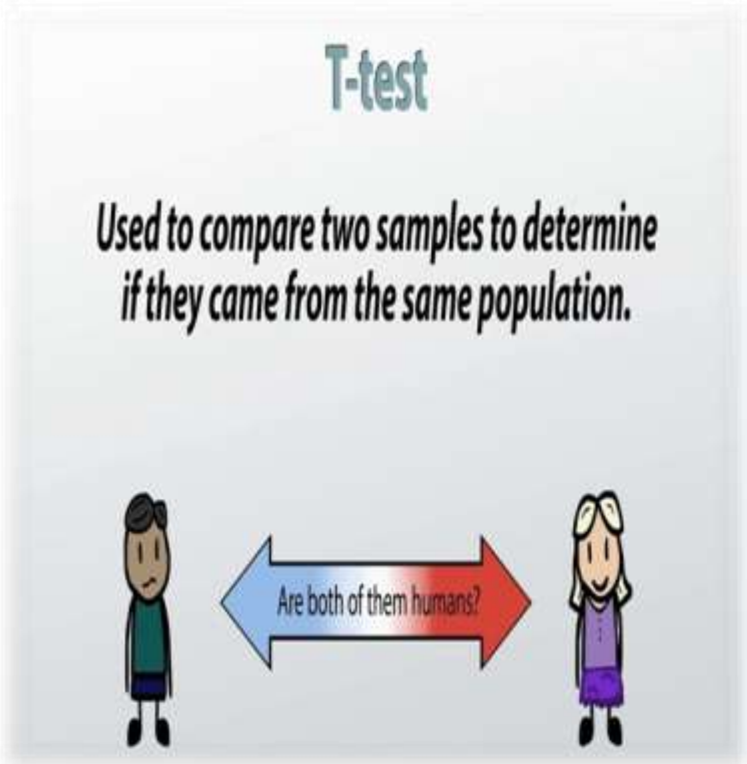
t-test

t –test is about means: distribution and evaluation for group distribution

Withdrawn form the normal distribution

The shape of distribution depend on sample size and, the sum of all distributions is a normal distribution

t- distribution is based on sample size and vary according to the degrees of freedom



What is the t -test

t test is a useful technique for comparing mean values of two sets of numbers.

The comparison will provide you with a statistic for evaluating whether the difference between two means is statistically significant.

t test is named after its inventor, William Gosset, who published under the pseudonym of student.

t test can be used either :

1. to compare two independent groups (independent-samples *t* test)
2. to compare observations from two measurement occasions for the same group (paired-samples *t* test).

What is the t -test

The null hypothesis states that any difference between the two means is a result to difference in distribution.

Remember, both samples drawn randomly form the same population.

Comparing the chance of having difference is one group due to difference in distribution.

Assuming that both distributions came from the same population, both distribution has to be equal.

What is the t -test

Then, what we intend:

"To find the difference due to chance"

Logically, The larger the difference in means, the more likely to find a significant *t* test.

But, recall:

1. Variability

More variability = less overlap = larger difference

2. Sample size

Larger sample size = less variability (pop) = larger difference

Types

1. The ***independent-sample t test*** is used to compare two groups' scores on the same variable. For example, it could be used to compare the salaries of dentists and physicians to evaluate whether there is a difference in their salaries.

2. The ***paired-sample t test*** is used to compare the means of two variables within a single group. For example, it could be used to see if there is a statistically significant difference between starting salaries and current salaries among the general physicians in an organization.

Assumption

1. Dependent variable should be continuous (I/R)
2. The groups should be randomly drawn from normally distributed and independent populations

e.g. Male X Female

Dentist X Physician

Manager X Staff

NO OVER LAP

Assumption

3. the independent variable is categorical with two levels
4. Distribution for the **two independent** variables is normal
5. Equal variance (homogeneity of variance)
6. large variation = less likely to have sig t test = accepting null hypothesis (fail to reject) = Type II error = a threat to power

Sending an innocent to jail for no significant reason

Independent Samples *t*-test

Used when we have two independent samples, e.g., treatment and control groups.

Formula is:

$$t_{\bar{X}_1 - \bar{X}_2} = \frac{\bar{X}_1 - \bar{X}_2}{SE_{diff}}$$

Terms in the numerator are the sample means.

Term in the denominator is the standard error of the difference between means.

Independent samples *t*-test

The formula for the standard error of the difference in means:

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}}$$

Suppose we study the effect of caffeine on a motor test where the task is to keep a the mouse centered on a moving dot. Everyone gets a drink; half get caffeine, half get placebo; nobody knows who got what.

Independent Sample Data (Data are time off task)

Experimental (Caff)	Control (No Caffeine)
12	21
14	18
10	14
8	20
16	11
5	19
3	8
9	12
11	13
	15
N_1=9, M_1=9.778, SD_1=4.1164	N_2=10, M_2=15.1, SD_2=4.2805

Independent Sample Steps(1)

1. Set alpha. Alpha = .05
2. State Hypotheses.

   Null is H_0: $\mu_1 = \mu_2$.

   Alternative is H_1: $\mu_1 \neq \mu_2$.

Independent Sample Steps(2)

3. Calculate test statistic:

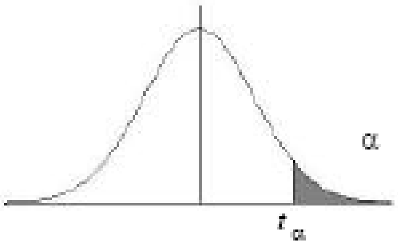
$$t = \frac{\overline{X}_1 - \overline{X}_2}{SE_{diff}} = \frac{9.778 - 15.1}{1.93} = \frac{-5.322}{1.93} = -2.758$$

$$SE_{diff} = \sqrt{\frac{SD_1^2}{N_1} + \frac{SD_2^2}{N_2}} = \sqrt{\frac{(4.1164)^2}{9} + \frac{(4.2805)^2}{10}} = 1.93$$

Independent Sample Steps (3)

4. Determine the critical value. Alpha is .05, 2 tails, and df = N1+N2-2 or 10+9-2 = 17. The value is 2.11.
5. State decision rule. If |-2.758| > 2.11, then reject the null.
6. Conclusion: Reject the null. the population means are different. Caffeine has an effect on the motor pursuit task.

Table 4: Percentage Points of the *t* distribution



df	α 0.250	0.100	0.050	0.025	0.010	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
⋮						
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
∞	0.674	1.282	1.645	1.960	2.326	2.576

Using SPSS

Open SPSS
Open file "SPSS Examples" for Lab 5
Go to:

- "Analyze" then "Compare Means"
- Choose "Independent samples t-test"
- Put IV in "grouping variable" and DV in "test variable" box.
- Define grouping variable numbers.
  - E.g., we labeled the experimental group as "1" in our data set and the control group as "2"

Independent Samples Exercise

Experimental	Control
12	20
14	18
10	14
8	20
16	

Work this problem by hand and with SPSS.

You will have to enter the data into SPSS.

SPSS Results

Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
TIME	experimental group	5	12.0000	3.1623	1.4142
	control group	4	18.0000	2.8284	1.4142

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
TIME	Equal variances assumed	.130	.729	-2.958	7	.021	-6.0000	2.0284	-10.7963	-1.2037
	Equal variances not assumed			-3.000	6.857	.020	-6.0000	2.0000	-10.7493	-1.2507

Dependent Samples t-tests

Dependent Samples *t*-test

Used when we have dependent samples – matched, paired or tied somehow

- Repeated measures
- Brother & sister, husband & wife
- Left hand, right hand, etc.

Useful to control individual differences. Can result in more powerful test than independent samples *t*-test.

Dependent Samples *t*

Formulas:

$$t_{\overline{X}_D} = \frac{\overline{D}}{SE_{diff}}$$

t is the difference in means over a standard error.

$$SE_{diff} = \frac{SD_D}{\sqrt{n_{pairs}}}$$

The standard error is found by finding the difference between each pair of observations. The standard deviation of these difference is SD_D. Divide SD_D by sqrt (number of pairs) to get SE_{diff}.

Another way to write the formula

$$t_{\overline{X}_D} = \frac{\overline{D}}{SD_D / \sqrt{n_{pairs}}}$$

Dependent Samples *t* example

Person	Painfree (time in sec)	Placebo	Difference
1	60	55	5
2	35	20	15
3	70	60	10
4	50	45	5
5	60	60	0
M	55	48	7
SD	13.23	16.81	5.70

Dependent Samples *t* Example (2)

1. Set alpha = .05
2. Null hypothesis: H_0: $\mu_1 = \mu_2$. Alternative is H_1: $\mu_1 \neq \mu_2$.
3. Calculate the test statistic:

$$SE_{diff} = \frac{SD}{\sqrt{n_{pairs}}} = \frac{5.70}{\sqrt{5}} = 2.55$$

$$t = \frac{\overline{D}}{SE_{diff}} = \frac{55 - 48}{2.55} = \frac{7}{2.55} = 2.75$$

Dependent Samples t Example (3)

4. Determine the critical value of t.
   Alpha =.05, tails=2
   df = N(pairs)-1 =5-1=4.
   Critical value is 2.776
5. Decision rule: is absolute value of sample value larger than critical value?
6. Conclusion. Not (quite) significant. Painfree does <u>not</u> have an effect.

Using SPSS for dependent t-test

Open SPSS

Open file "SPSS Examples" (same as before)

Go to:

- "Analyze" then "Compare Means"
- Choose "Paired samples t-test"
- Choose the two IV conditions you are comparing. Put in "paired variables box."

Dependent t- SPSS output

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PAINFREE	55.0000	5	13.2288	5.9161
	PLACEBO	48.0000	5	16.8077	7.5166

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	PAINFREE & PLACEBO	5	.956	.011

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	PAINFREE - PLACEBO	7.0000	5.7009	2.5495	-7.86E-02	14.0786	2.746	4	.052

Relationship between t Statistic and Power

$$t = \frac{\overline{X} - \mu}{SE_M}$$

To increase power:

- Increase the difference between the means.
- Reduce the variance
- Increase N
- Increase α from α = .01 to α = .05

To Increase Power

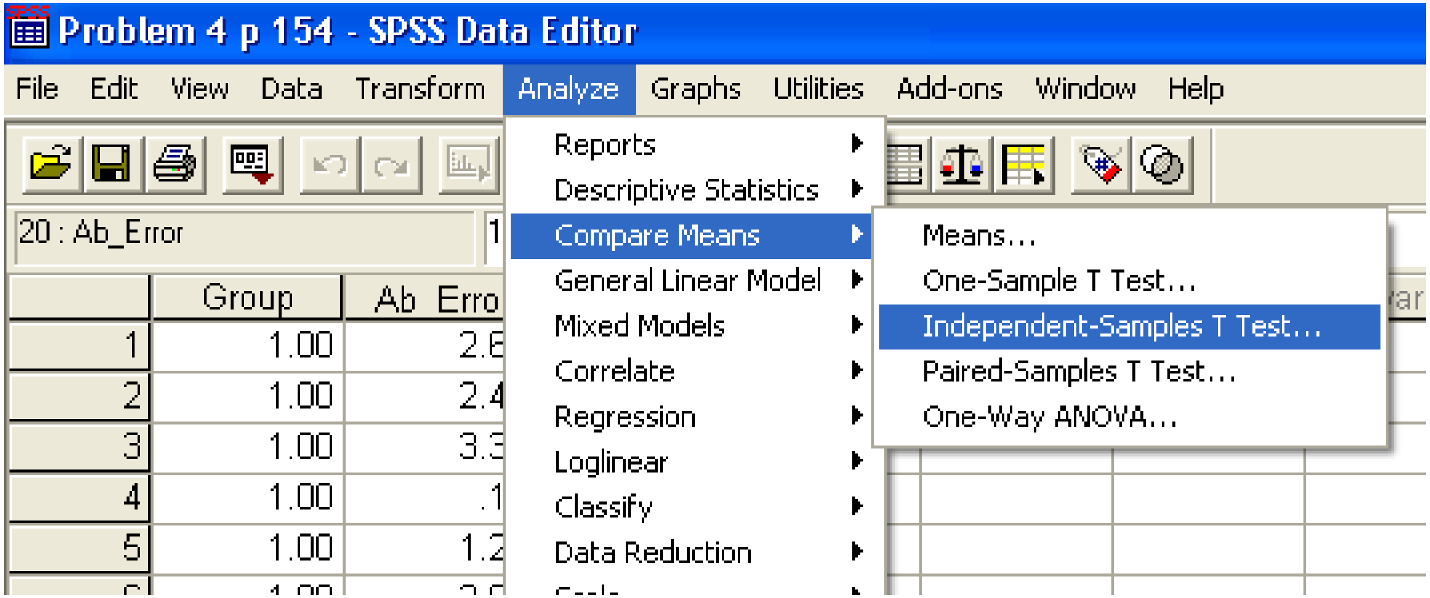
Increase alpha, Power for $\alpha = .10$ is greater than power for $\alpha = .05$

Increase the difference between means.

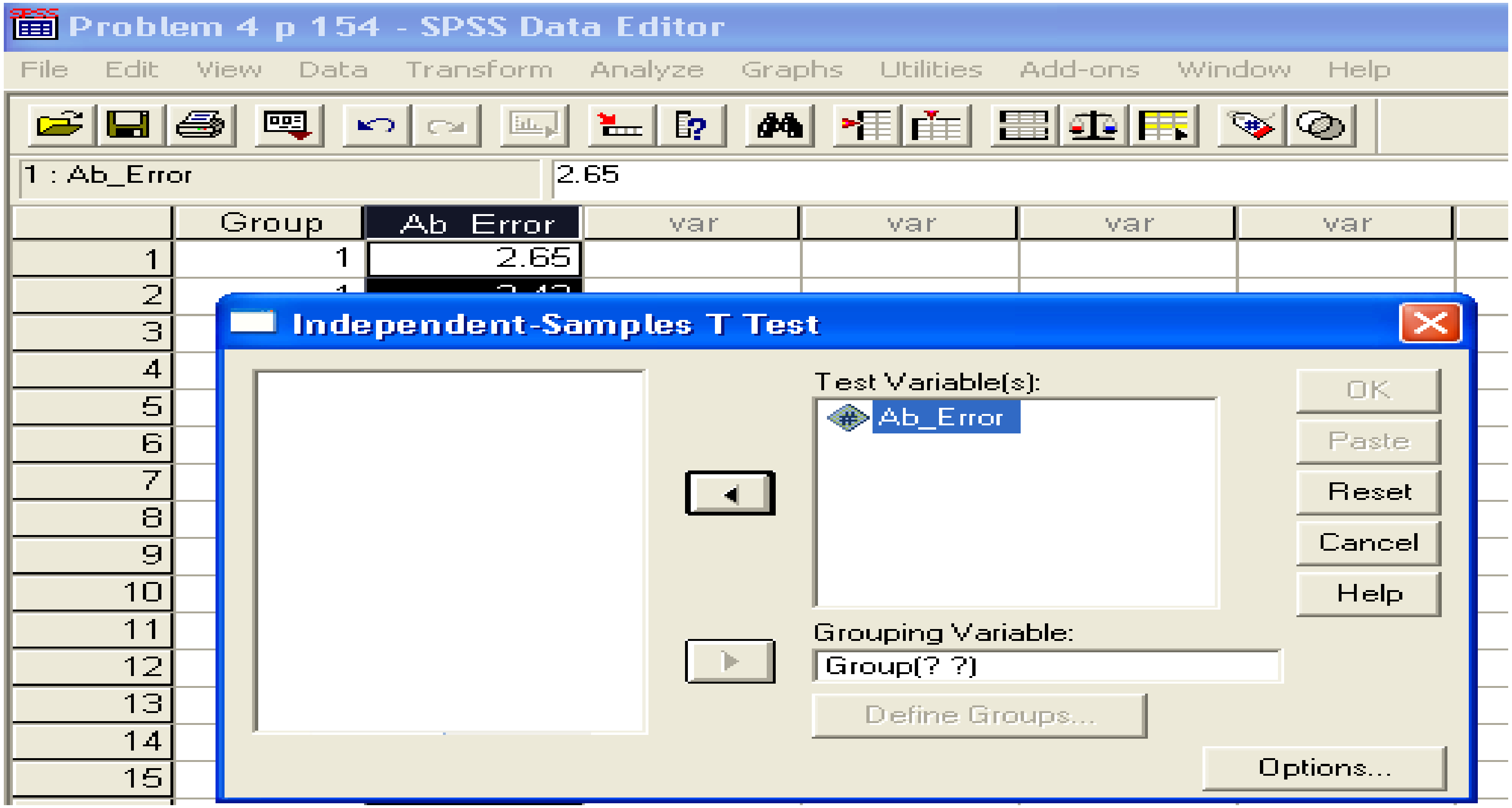
Decrease the sd's of the groups.

Increase N.

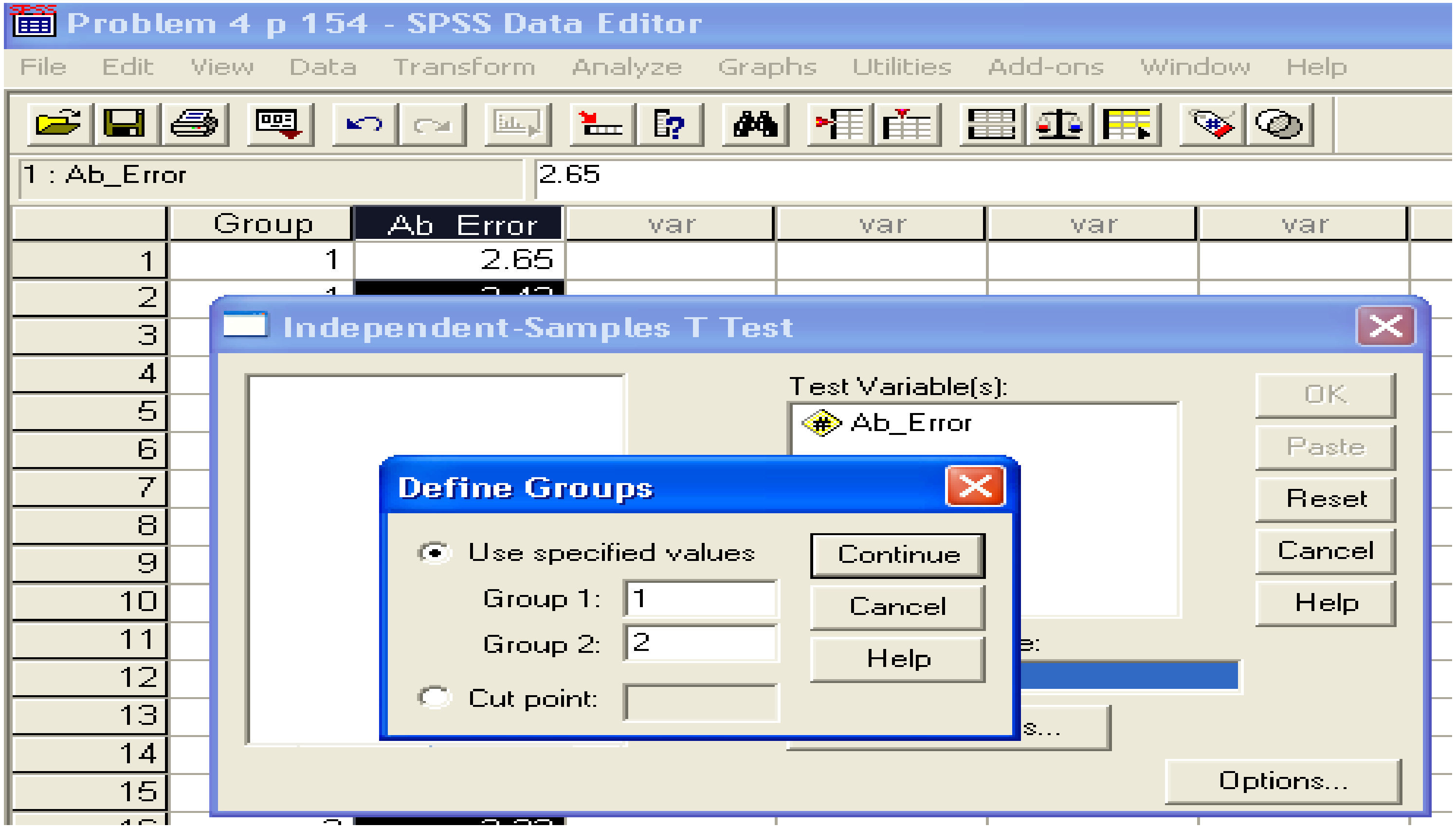
Independent t-Test



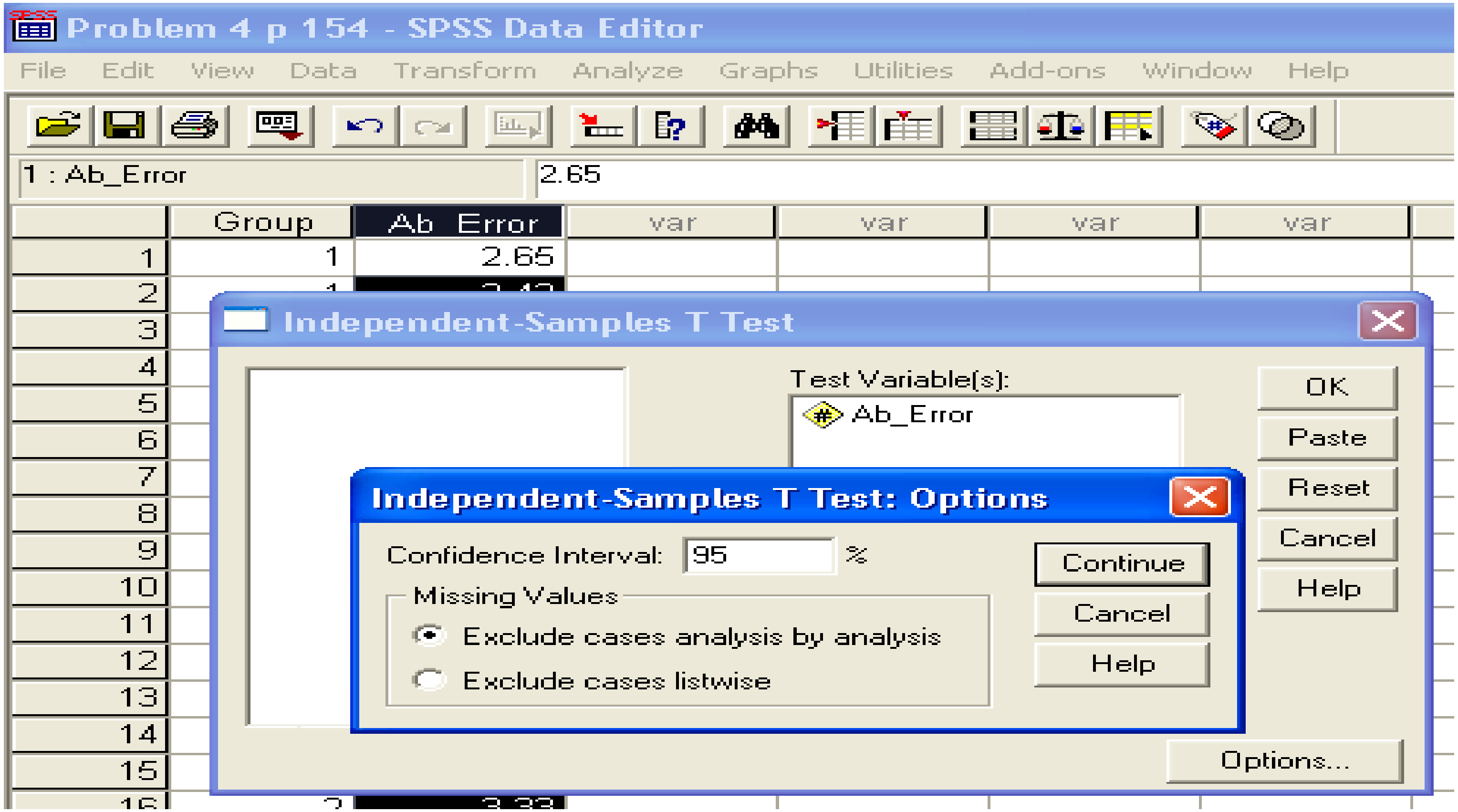
Independent t-Test: Independent & Dependent Variables



Independent t-Test: Define Groups



Independent t-Test: Options



Independent t-Test: Output

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
Ab_Error	Active	10	2.2820	1.24438	.39351
	Passive	10	1.9660	1.50606	.47626

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Ab_Error	Equal variances assumed	.513	.483	.511	18	.615	.31600	.61780	-.98194	1.61394
	Equal variances not assumed			.511	17.382	.615	.31600	.61780	-.98526	1.61726

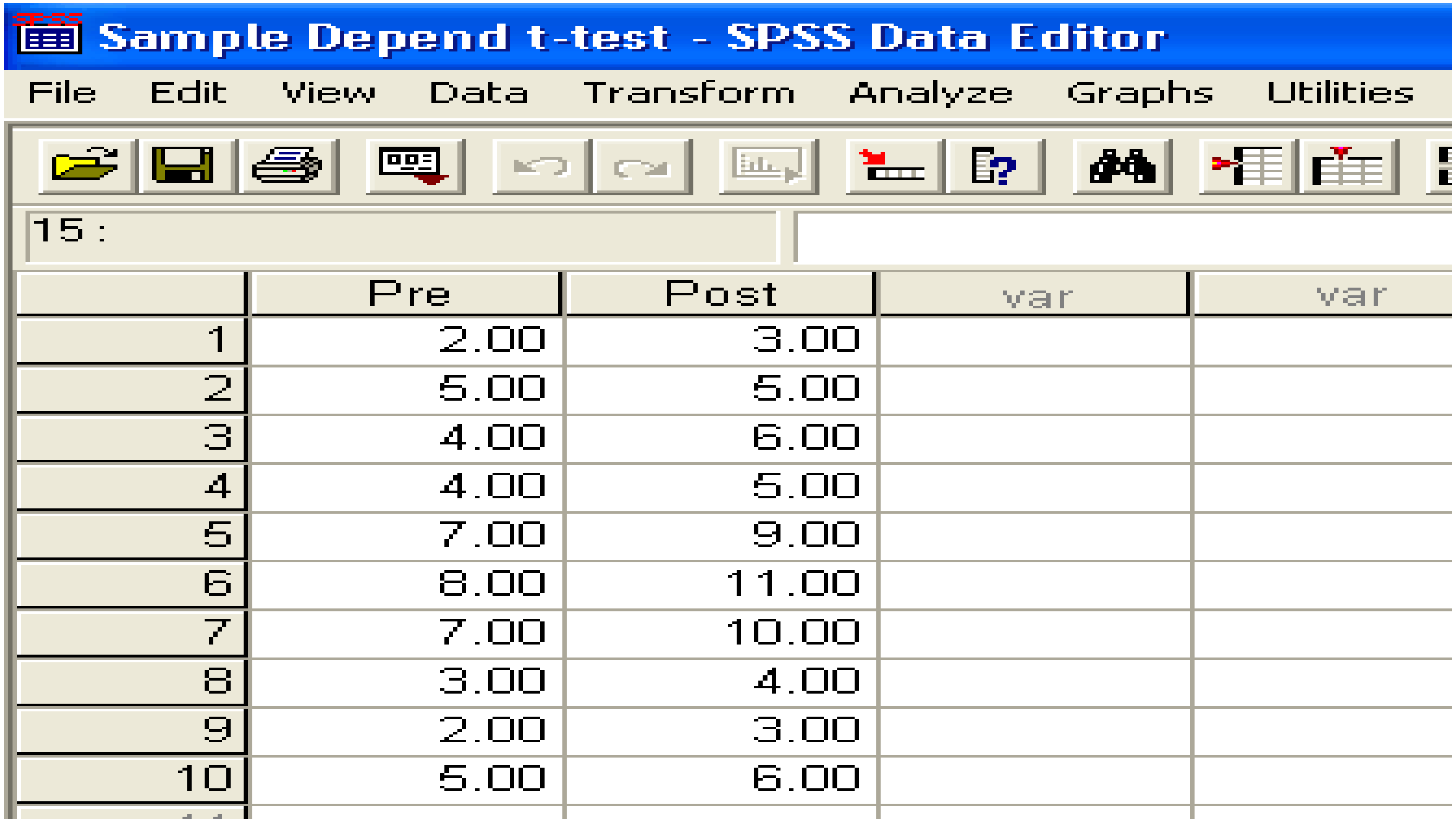
Are the groups different?

$t(18) = .511, p = .615$

NO DIFFERENCE

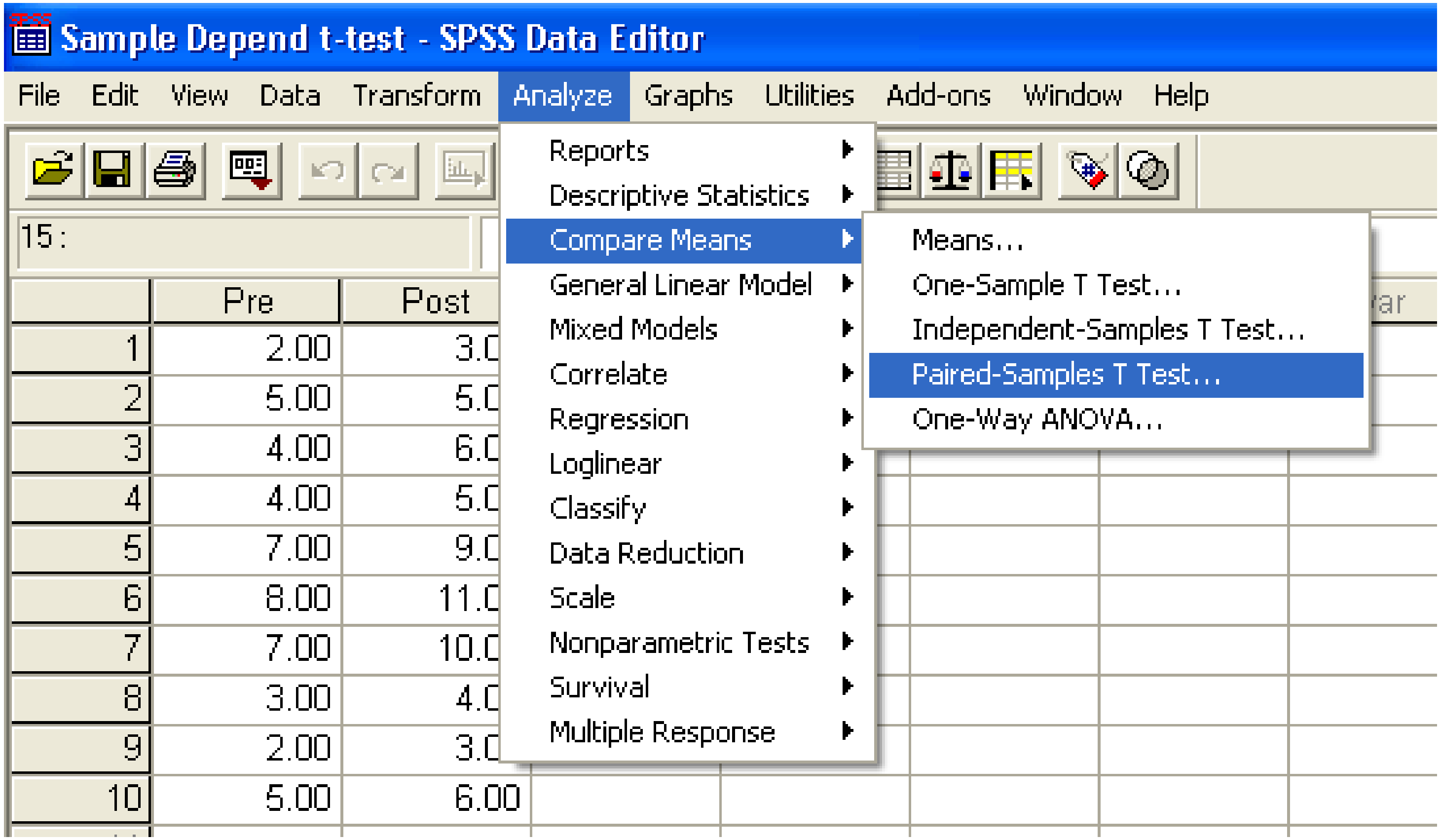
2.28 is not different from 1.96

Dependent or Paired t-Test: Define Variables

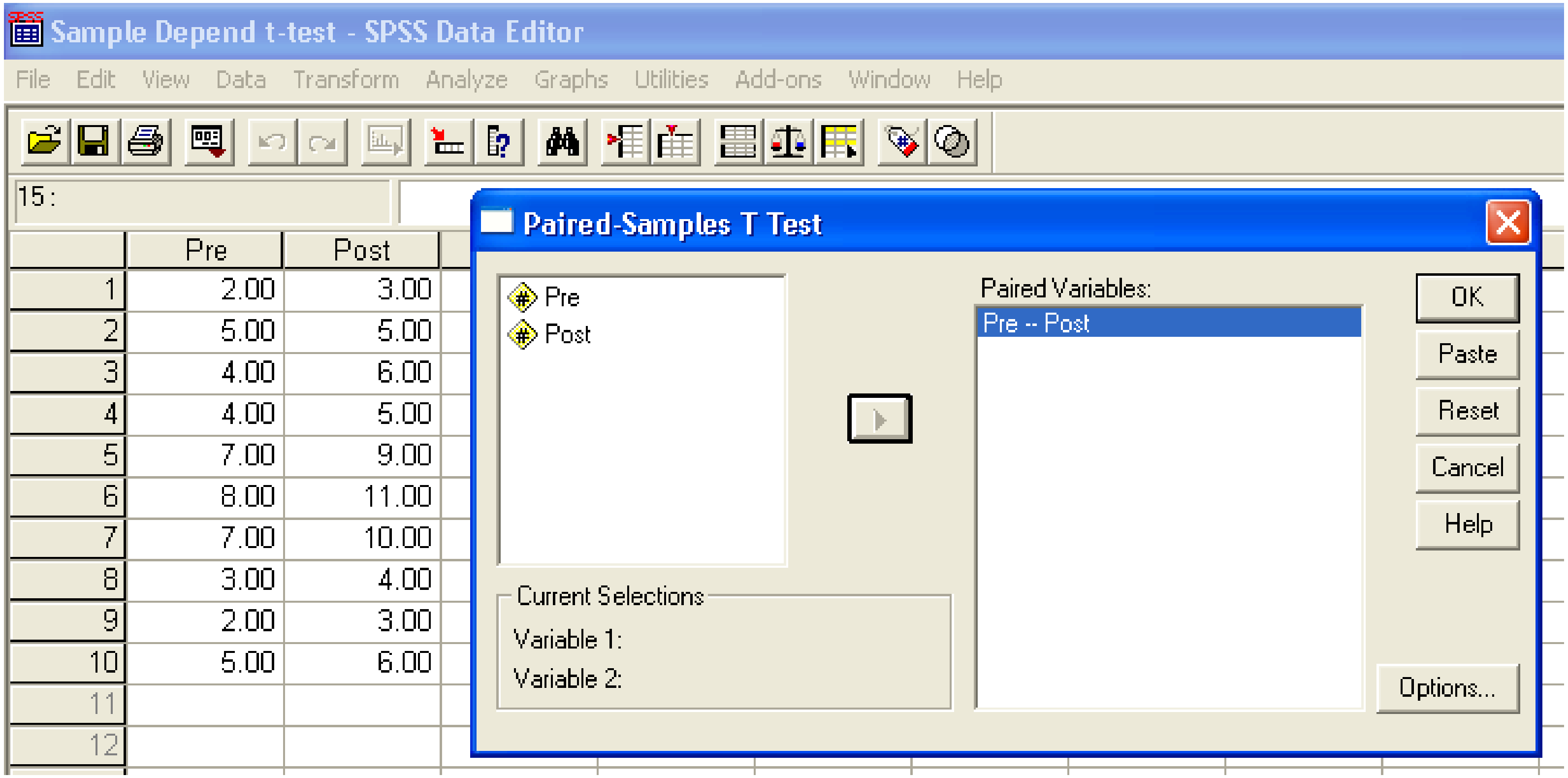


	Pre	Post	var	var
1	2.00	3.00		
2	5.00	5.00		
3	4.00	6.00		
4	4.00	5.00		
5	7.00	9.00		
6	8.00	11.00		
7	7.00	10.00		
8	3.00	4.00		
9	2.00	3.00		
10	5.00	6.00		

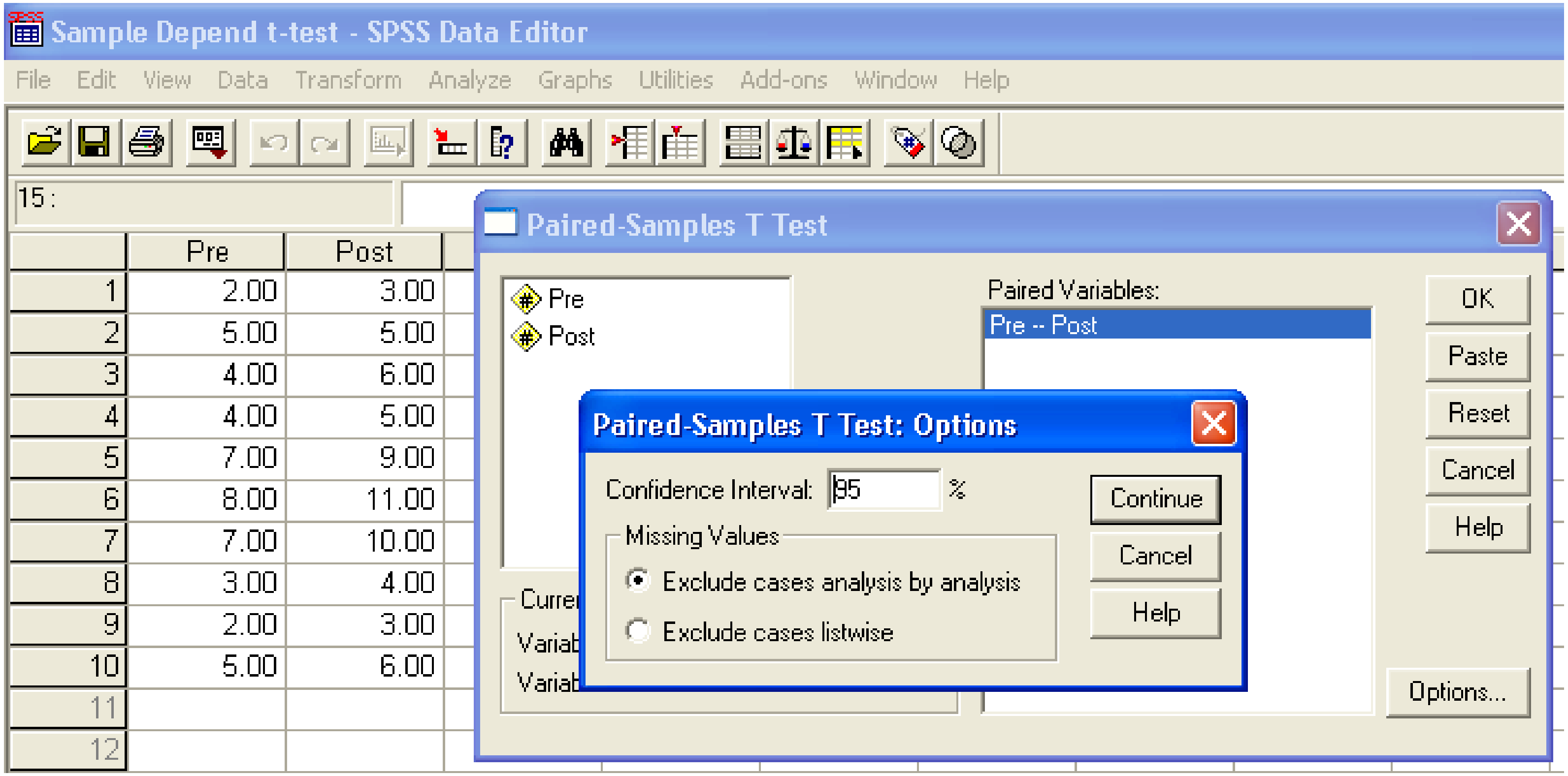
Dependent or Paired t-Test: Select Paired-Samples



Dependent or Paired t-Test: Select Variables



Dependent or Paired t-Test: Options



Dependent or Paired t-Test: Output

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre	4.7000	10	2.11082	.66750
	Post	6.2000	10	2.85968	.90431

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Pre & Post	10	.968	.000

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	95% Confidence Interval of the Difference: Upper			
Pair 1	Pre - Post	-1.50000	.97183	.30732	-2.19520	-.80480	-4.881	9	.001

Is there a difference between pre & post?

$t(9) = -4.881, p = .001$

Yes, 4.7 is significantly different from 6.2